Strategies Used to Promote Discourse and Engagement in Mathematics Classrooms

“We Learn . . .
10% of what we read
20% of what we hear
30% of what we see
50% of what we see and hear
70% of what we discuss
80% of what we experience
95% of what we teach others."  William Glasser,

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## Study Team and Teaching Strategies (STTS)

<table>
<thead>
<tr>
<th>SPARC</th>
<th>TEAMS</th>
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<tbody>
<tr>
<td>Start promptly.</td>
<td>Together, work to answer questions.</td>
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<tr>
<td>Peer support expected within each team.</td>
<td>Explain and give reasons.</td>
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<td>Assignments due each day.</td>
<td>Ask questions and share ideas.</td>
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<tr>
<td>Respond to group rather than individuals.</td>
<td>Members of your team are your first resource.</td>
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### Carousel: Around the World
- Write a different problem/topic/question on large poster sheets hung on the walls or on each table.
- Each team is given a different colored marker.
- Each team goes to a different poster, discusses the topic and decides what to write.
- Teams rotate to all of the posters, adding to what was written by previous teams (have a time limit).
- When done, each team does a “gallery walk.”
- A large group discussion/debrief can then be held.

### Carousel: Index Card
- Put the problem on regular paper or cardstock.
- The card gets passed around. It can be written on or post-its could be added to the back.

### Carousel: Station Rotation
- Have 1-2 more stations than the number of student groups.
- Place a sheet of review problems (4-6) at each station.
- Have a blank answer sheet at each station for each group.
- The students work the problems as a group when they finish they turn in the station paper to the teacher and move to the next available station.

### Dyad
- Each person is given equal time to talk.
- The listener does not talk; it isn’t a conversation.
- Confidentiality is maintained.
- Maintain eye contact and good body language.

### Elevator Talk
- Each person/team is given a topic.
- They summarize the topic into a quick presentation.
Fishbowl
• Used to model to whole class expected behaviors/norms.
• One or two teams sit in the middle of the class and works on the math problem.
• Rest of class stands near the team and observes or takes notes of how the team works, questions that are asked…..
• After 5-10 minutes, the teams return to their own tables and work on the math problem.

Fortune Cookie
• Choose 5-6 questions and put in an envelope.
• Each team receives an envelope.
• One person draws a question, and makes one statement about the topic, then passes it on.
• The next person adds his or her own statement or responds to the previous statement.
• When everyone has responded to the first statement, another person draws from the envelope and repeats the process.

Gallery or Museum Walk
• Students post their presentations around the room.
• Students, individually or in teams, walk around and look at the presentations.
• Students give feedback.

Give One Get One
• Record three ideas to share related to a certain topic.
• Circulate and share ideas; for every idea given they receive one in return and record these on a piece of paper – including the name of the author.
• Begin team sharing by inviting a volunteer to share one idea received citing the author. The named person then continues the sharing process.

Hot Potato~ (Round Table)
• Every team has one sheet of paper and each student has a different colored pencil.
• A problem is given to the group and placed in the middle of the table.
• Student 1 writes the first step of the solution process, explaining aloud, and passes the paper on to Student 2.
• Student 2 makes any corrections and adds the next step, explaining aloud, and passes the paper on.
• Process continues until the problem is competed.
**Hot Seat**
- One chair/desk per team is set up in the front of the room.
- Using Numbered Heads, Student #1 from each team comes to the front of the room and sits.
- Teacher gives everyone a problem to work on in a specified amount of time.
- Teams can talk, but not the individuals in front.
- Check individual and team answers; two points for correct individual answers and 1 point for correct team answers.
- Student #2 from each team is up next and repeat.

**Huddle**
- One person from each team (teacher’s choice) is called to the front of the room.
- Teacher gives a piece of information, checks for understanding….
- Student goes back to team to share.

**I Spy**
- When the team is stuck, one student (teacher’s choice) can go around to another team and listen in.
- No talking.
- Student reports back to the team what was learned.

**Jigsaw**
- Each study team member is assigned a different part of a task/topic.
- Each member researches/learns about the task/topic (possibly with others with same topic).
- Each member then presents the information to the others in his/her study team.

**Listening Post**
- Students #1 and #2 work on a math problem aloud in their team.
- Student #3 listens to the discussion and can ask clarifying math questions.
- Student #4 only records what is discussed and verbalized (looks for attitudes) and may not talk.
- After 15 minutes, work stops and student #4 shares notes and observations.
- A variation is Students #1, #2, and #3 work and #4 observes and then shares.

**Math Chat**
- Have posters, with a topic on each one.
- Each person has a writing utensil.
- No talking.
- People write something about the topic.
- When it’s done, it’s done.

**Numbered Heads**
- Students number off in study team.
- The team is given a problem to do.
- When the team finishes, use random numbers (1-4) to ask questions or have team members share the solution process.
- The numbers can also be used to assign roles.
Pairs Check~(Rally Coach)  
- Each pair has one paper and pencil.  
- Student #1 writes what Student #2 explains.  
- Then roles are reversed for the second problem.  
- Then each pair checks their work with the other study team pair.  
- Continue on to the next pair of problems.

Participation Quiz  
- Pick a group worthy task.  
- Tell students which norm you are focusing on.  
- Show teams how you are keeping track (overhead, posters, chalkboard).  
- Record comments while students are working.  
- Debrief (Do not need to record everything).

Proximity Partner:  
- Students stand up and move to find a partner.  
- They share information with their partner.  
- They return to their team.

Peer Edit  
- Students write.  
- Peers read aloud or switch papers.  
- Peers edit the paper (orally or in writing).  
- Return to the writer for rewrite.

Reciprocal Teaching  
- In pairs, Student A pretends that Student B was absent and explains a concept.  
- Switch roles and continue.

Red Light, Green Light  
- The team works together on a problem or set of problems.  
- When they finish the problem, then they must Stop.  
- The teacher verifies the work/answer with questions.  
- The team is then given permission to Go to the next problem or set of problems.

Silent Debate  
- Student pairs: One is “pro,” the other “con.”  
- Each pair has one pencil and one sheet of paper.  
- A topic is given and the pro goes first.  
- The pro makes a supportive statement in writing.  
- The con reads the statement and then writes a comment against the topic.  
- The process repeats 3-4 times.

Swapmeet  
- When a group task is partially finished, one pair from each team rotates to the next team.  
- Pairs from the two teams share ideas, solutions, thinking…  
- Pairs return to their original teams and share what they learned.
Teammates Consult (Pencils in the Middle)

- All pencils and calculators are set aside.
- Students read the problem or question.
- Give students individual think/work time.
- Teams discuss the problem for clarity.
- Possible strategies are shared.
- Teacher gives okay for pencils to be picked up and written work to begin.

Think-Ink-Pair-Share~ (Think-Pair-Share)

- Teacher poses a question/problem.
- Without pencils, students think for 1-2 minutes.
- Students may then use pencil to begin working…without talking to partner.
- Students then share their thinking and answer(s) with their partner.
- Pairs then may share with larger group.

Traveling Salesman

- Teacher assigns a topic/problem to each team.
- Students solve the problem then plan a presentation.
- One team member presents the mathematics to another team.
- Repeat.

Tuning Protocol

- One person presents problem to team.
- Teammates ask clarifying questions.
- Presenter turns around.
- Teammates discuss problem, coming to a better understanding.
- Presenter takes notes and reflects on what is said.

Walk and Talk

- Topic is presented.
- Pairs walk around classroom (or meet with a partner) discussing the topic.

Whiparound

- Topic or question is presented.
- Participants randomly have an opportunity to say something briefly about it.
- Everyone does not have to comment but are encouraged to do so. Each student has one card with problem and an answer to a different problem.

Whiparound (I Have…., Who Has….)

- Student 1 asks “Who has…” and states the problem.
- The person with the solution says “I have ….” and states the answer.
- The responding student then poses his problem and the student with the answer on his card responds.
- The process continues until all the questions and responses are given.

Resources: http://www.cpm.org/teachers/study.htm
Student Roles in Study Teams:

Assigning students particular roles in their teams can support productive teamwork. The purpose of the roles is to give each student a clear way of participating in the team conversation. Roles also allow students to share responsibility for the effective functioning of the team and class.

While specific strategies are outlined here and in teacher notes for key lessons to help the teacher implement defined team roles, the textbook is written so that a teacher may choose whether or not to use these roles. The student text makes no mention of team roles after the first section of the first chapter, so it is up to the teacher to use the team role resource pages (displayed on an overhead or document camera, or distributed to teams) to help each student learn his or her role.

Team roles can be structured in a variety of ways. We suggest assigning students the following roles when working in teams of four: Resource Manager, Facilitator, Recorder/Reporter, and Task Manager. These roles are further described below.

Ideally, students will have the opportunity to serve in each role over the course of the term. Some teachers will want to assign students roles that last for a week or for the entire time the student is with a particular team. Other teachers may wish to change students’ roles more frequently. It is recommended that teachers assign roles randomly. Some teachers post roles on class seating charts, while others assign them to specific seats within each team (for instance, by using colored dots on the table corners).

In order for the roles to support learning successfully, students need to see that these roles have value in study team interactions. Teachers must stick to and emphasize the roles over time so that students have ample opportunities to learn how to perform different roles. Many students will benefit from hearing sample statements that illustrate what their role might sound like in action. Teacher notes in the first section provide suggestions for how to assign team roles in the context of each activity. Similar suggestions are offered throughout the course, and we encourage teachers to look for additional opportunities to make the most of team roles.
Teaching “Team Roles” to Your Study Teams Handout

Each role should be on a different colored sheet. Cut apart, collate in groups of four different colored team roles and distribute them to your regular study teams. (Now each person within the team will have a different colored card!)

Jigsaw the initial roles by having all the same-color, same role persons report to one of the four different corners of the classroom. For example, all Recorder/Reporters report to the same corner to discuss how they see themselves handling this role, what questions they will need to ask their team-members, and what behaviors they will encourage within their teams. Take only a short time on this and have them reconvene in their regular study teams to share their roles out with the other roles.

Summarize with a whole-class discussion about these roles so that everyone hears the roles for the first time and specific students begin to learn their roles. For example, “Raise your hand if your job is to be the Resource Manager for today?” As teacher, you can easily scan the class and know if the correct students are responding. “What is one question you will ask your group members today?” Go through all four “jobs” in this manner.

Launch a group-worthy task from your course that you will use to introduce teach them the team roles. Remember in all connections courses there are specific problems in which the team roles cards have been adapted to give students more appropriate questions to ask for that task.

Cruise the groups, jotting down on your clip board (tablet) examples of the appropriate questions you hear the role members asking as they solve this task. At first they will probably use only the exact questions from the role cards. Process this with them at the end of the session reinforcing the positive questions you have heard and how the team roles help make the groups work more efficiently. For example: “How does it help your groups to work more efficiently if your members do these specific jobs?”

During the next three days, have them keep the same roles while continuing to deepen the processing questions as you progress through the classwork problems in chapter one. Say, for example: “You didn't hear all the groups like I did as I walked around, but students are even starting to ask similar questions within these roles!” Support this by reading their questions that you heard.

During the next two weeks, rotate the jobs within the team so students really have a chance to learn and practice each role. The processing time shortens but the importance of the roles and how they help the team to function is kept in front of them daily during the first weeks of school! While debriefing during the later part of these two weeks, display the team roles norms and discuss each one together in class. Ask them to state the evidence they have observed supporting how these norms help teams to function more smoothly.

Continue throughout the first quarter and the school year to work relentlessly to make the study teams improve in their effectiveness. Developing this smooth teamwork makes using a student-centered classroom work to its full potential while allowing students to gain a deeper conceptual understanding of the mathematics.
<table>
<thead>
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<th><strong>Resource Managers</strong></th>
<th><strong>Task Manager</strong></th>
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<td>get necessary supplies and materials for the team and make sure that the team has cleaned up its area at the end of the day. They also manage the non-material resources for the team, seeking input from each person and then calling the teacher over to ask a team question. Typically, a teacher could expect to hear a resource manager asking:</td>
<td>keeps the team focused on the assignment of the day. He or she works to keep the team discussing the math at hand and monitors if anyone is talking outside of her/his team. Additionally, a task manager helps the team focus on articulating the reasons for the math statements they make. Typically, a teacher could expect to hear a task manager saying:</td>
</tr>
<tr>
<td>“Does anyone have an idea?”</td>
<td>“Ok, let’s get back to work!”</td>
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<tr>
<td>“Who can answer that question? Should I call the teacher?”</td>
<td>“Let’s keep working.”</td>
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<tr>
<td>“What supplies do we need for this activity?”</td>
<td>“What does the next question say?”</td>
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<tr>
<td><strong>Recorder/Reporters</strong></td>
<td><strong>Facilitators</strong></td>
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<td>share the team’s results with the class (as appropriate) and serve as a liaison with the teacher when s/he has additional information to share with the class and calls for a “huddle” with all of the recorder/reporters. In some activities, a recorder/reporter may make sure that each team member understands what information s/he needs to record personally. Recorder/reporters may also take responsibility for organizing their team members’ contributions as they prepare presentations. Typically, a teacher could expect to hear a recorder/reporter asking:</td>
<td>help their teams get started by having someone in the team read the task aloud. They make sure each person understands the task and that the team helps everyone know how to get started. Before anyone moves on, the facilitator asks to make sure each team member understands the team’s answer. Typically, a teacher could expect to hear a facilitator asking:</td>
</tr>
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<td>“Does everyone understand what to write down?”</td>
<td>“Who wants to read?”</td>
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<tr>
<td>“How should we show our answer on this poster?”</td>
<td>“Does anyone know how to get started?”</td>
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<tr>
<td>“Can we show this in a different way?”</td>
<td>“What does the first question mean?”</td>
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<td>“What does each person want to explain in the presentation?”</td>
<td>“I’m not sure - What are we supposed to do?”</td>
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<td>“I’m not sure - What are we supposed to do?”</td>
<td>“Can we show this in a different way?”</td>
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<tr>
<td>“Do we all agree?”</td>
<td>“What does each person want to explain in the presentation?”</td>
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<tr>
<td>“I’m not sure I get it yet - can someone explain?”</td>
<td>“Tell me why!”</td>
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To maximize their learning opportunities, students are expected to actively participate within their study teams. To create this norm in the class, it is important to begin teaching students your expectations for effective teamwork from the beginning. Activities and lesson structures suggested in the teacher notes, resource pages, and descriptions of team roles begin to communicate expectations to students in Chapter 1 (see description below). So as not to interfere with teachers’ setting their own class norms, these guidelines only appear once in the student text in Section 1.1. Nevertheless, we suggest the following guidelines for teams which can be remembered by using the acronym TEAMS:

- **Together, work to answer questions.**
  - Requiring students to work within their team helps them to see each other as resources and to find their own way of solving a problem. By making students look to the others in their team rather than friends in other parts of the classroom, it helps ensure that no student is excluded from the conversation. Teams should work and move as a team without leaving anyone behind or having anyone working ahead. Emphasizing the importance of creating space to share ideas and converse openly about the mathematics will help teams be more cohesive.

- **Explain and give reasons.**
  - This norm links directly the learning themes of this course and underscores the expectation that there are multiple valid ways of solving a problem.

- **Ask questions and share ideas.**
  - This helps to set a tone that the classroom is a community of support. This expectation also challenges students to help a teammate understand and make sense of ideas for him- or herself instead of simply receiving an answer. It also reminds students that their conversations in study teams have an intellectual, rather than a social, purpose.

- **Members of your team are your first resource.**
  - This norm can be reinforced by the manner in which the teacher responds to questions from a team. Responding only to the hand of the resource manager and then asking, “Does everyone in the study team want the question answered?” helps students to work on answering their own questions. This norm should not imply that the teacher does not answer questions, but instead that the other members of the team are a student’s first resource. While this behavior can be as difficult for the teacher as it is for the student, it clearly and concretely teaches students to become self-directed learners.

- **Smarter together than apart.**
  - Again, this norm emphasizes that the solving process and thinking mathematically are important parts of every problem, and that understanding others’ approaches improves an individual’s understanding.
Decide what information the $x$-axis and $y$-axis could represent so that each point represents a different member of your team.
YOU ARE GETTING SLEEPY…

Legend has it that if you stare into a person’s eyes in a special way, you can hypnotize them into squawking like a chicken. Here’s how it works.

Place a mirror on the floor. Your victim has to stand exactly 200 cm away from the mirror and stare into it. The only tricky part is that you need to figure out where you have to stand so that when you stare into the mirror, you are also staring into your victim’s eyes.

If your calculations are correct and you stand at the exact distance, your victim will squawk like a chicken!

a. Choose a member of your team to hypnotize. Before heading to the mirror, first analyze this situation. Draw a diagram showing you and your victim standing on opposite sides of a mirror. Measure the heights of both yourself and your victim (heights to the eyes, of course), and label all the lengths you can on the diagram. (Remember, your victim will need to stand 200 cm from the mirror.)

b. Are there similar triangles in your diagram? Justify your conclusion. (Hint: Remember what you know about how light reflects off mirrors.) Then calculate how far you will need to stand from the mirror to hypnotize your victim.

c. Now for the moment of truth! Have your teammate stand 200 cm away from the mirror, while you stand at your calculated distance from the mirror. Do you make eye contact? If not, check your measurements and calculations and try again.
Red Light, Green Light

Are ratios only used to compare shapes that have been enlarged or reduced? In this lesson, you will expand your use of ratios to new situations. “What is being compared?” is a question that will be useful to keep in mind as you work with your team on this lesson.

4-75. Katura was making berry drink from a bag of powdered mix. The directions said to use 5 scoops of the powder for every 8 cups of water. Her pitcher holds 12 cups of water.

a. What is the ratio of powder to water in the directions?

b. Work with your team to figure out how much powder Katura needs to mix with 12 cups of water. Try to find more than one way to describe or show how you know that your answer makes sense. Be prepared to explain your ideas to the class.

c. What is the ratio of powder to water in Katura’s pitcher? How does this compare to the ratio in the directions?

4-76. ON THE TRAIL AGAIN

Ms. Hartley’s students were working with their mix of raisins and peanuts from Chapters 1 and 2. The class found that 30% of the mix was raisins. Sophie was working with a sample from the mix and counted 42 peanuts in it.

Sophie had just poured her sample back into the jar, when she realized that she had counted the wrong thing! Her teacher wanted to know how many raisins were in the sample, not peanuts! Work with your team and use the questions below to help Sophie figure out a reasonable estimate of how many raisins were in her sample.

a. Sophie knows that 30% is the same as \( \frac{30}{100} \). Can this be thought of as a ratio? Which two quantities are being compared in this case? Can you write another equivalent ratio representing the same comparison?

b. Could Sophie write a ratio comparing the number of raisins to peanuts? How could you figure out this ratio without having to count the peanuts? Discuss this with your team and be ready to explain your thinking to the class.

c. Find an equivalent ratio that will help Sophie figure out how many raisins should have been in her sample that contained 42 peanuts.
4-77. Nicci is setting up a carnival machine with 3 teddy bears, 7 stuffed frogs, 3 rubber duckies, and 2 stuffed dinosaurs.

a. Find the following ratios for Nicci’s machine:

i. The number of teddy bears to total prizes.

ii. The number of teddy bears to the number of stuffed dinosaurs.

iii. The number of teddy bears to the combined number of other prizes.

b. In the carnival game, one prize is chosen at random. Nicci’s teacher told her that the probability of randomly picking a teddy bear was 20%. Which of the ratios in part (a) do you think her teacher used to find the probability?

c. Nicci is setting up a different machine that holds 60 total prizes. The machine will have the same ratios for each kind of prize as her first machine. If the new machine has 12 teddy bears, will the chances of randomly picking a teddy bear be the same as for her original machine? Explain.

4-78. Trei correctly spelled 60% of the words on her last spelling test!

a. How many words did she spell correctly for each word that she spelled wrong? That is, what is her ratio of correctly to incorrectly spelled words?

b. Luis spelled 3 words correctly for every 1 that he spelled incorrectly. Did Luis do better than Trei on the test? What is Luis’s score represented as a percent?

4-79. Additional Challenge: A box is filled with green marbles, red marbles, and blue marbles. The ratio of red marbles to green marbles is 3:1. The ratio of green marbles to all of the marbles in the box is 2:11. Write each of the following ratios.

a. The ratio of red marbles to the total number of marbles.

b. The ratio of blue marbles to the total number of marbles.

c. The ratio of blue marbles to green marbles.

d. The ratio of red marbles to blue marbles.
#1

**BASE, EXPONENT, AND VALUE**

In the expression $2^5$, 2 is the **base**, 5 is the **exponent**, and the **value** is 32.

$2^5$ means $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

$x^3$ means $x \cdot x \cdot x$

Write each of these expressions as simply as possible using the method shown below.

Knowing that $x^3 = x \cdot x \cdot x$
then:

$$x(x^3) = x \cdot (x \cdot x \cdot x) = x^4$$

Use exponents to write each of the following expressions as simply as possible. Look for patterns as you do this with your study team. Write out the variables to show the meaning whenever necessary.

Write these out the long way, like the example.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>a) $x^2 \cdot x$</td>
<td>h) $x^3 \cdot x^6$</td>
</tr>
<tr>
<td>b) $y^2 \cdot y^5$</td>
<td>i) $x^3 \cdot x^2$</td>
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Write these out using the pattern or shortcut that you found.

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<tr>
<td>c) $(x^2)(x^5)$</td>
<td>j) $x^3 \cdot x^4$</td>
</tr>
<tr>
<td>d) $x^7 \cdot x^5$</td>
<td>k) $m^{13} \cdot m^{14}$</td>
</tr>
<tr>
<td>e) $y^8 \cdot y^6$</td>
<td>l) $x^{32} \cdot x^{59}$</td>
</tr>
<tr>
<td>f) $y^7 \cdot y^4$</td>
<td>m) $x^{31} \cdot x^{29}$</td>
</tr>
<tr>
<td>g) $x^3 \cdot x^5 \cdot x^4$</td>
<td>n) $y^3 \cdot y \cdot y^4$</td>
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Write the rule for multiplying exponential numbers in your own words.
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In the expression $2^5$, 2 is the **base**, 5 is the **exponent**, and the **value** is 32.

- $2^5$ means $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
- $x^3$ means $x \cdot x \cdot x$

Write each of these expressions as simply as possible using the method shown below.

Knowing that $x^3 = x \cdot x \cdot x$ then:

$$(x^3)^4 = (x \cdot x \cdot x)(x \cdot x \cdot x)(x \cdot x \cdot x)(x \cdot x \cdot x) = x^{12}$$

Use exponents to write each of the following expressions as simply as possible. Look for patterns as you do this with your study team. Write out the variables to show the meaning whenever necessary.

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<th>h) $(x \cdot y)^2$</th>
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<tr>
<td>b) $(y^2)^3$</td>
<td>i) $(x^2 \cdot y^3)^3$</td>
</tr>
<tr>
<td>c) $(x^5)^5$</td>
<td>j) $(2x)^4$</td>
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Write these out using the pattern or shortcut that you found.

<table>
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<th>d) $(x^3)^6$</th>
<th>k) $(x \cdot y)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>e) $(x^3)^9$</td>
<td>l) $(2x^2)^3$</td>
</tr>
<tr>
<td>f) $(x^4)^5$</td>
<td>m) $(x^2y^2)^4$</td>
</tr>
<tr>
<td>g) $(3^5)^4$</td>
<td>n) $(5^2)^4$</td>
</tr>
</tbody>
</table>

Write the rule for raising exponential numbers to a power in your own words.
#3

**BASE, EXPONENT, AND VALUE**

In the expression $2^5$, 2 is the **base**, 5 is the **exponent**, and the **value** is 32.

$2^5$ means $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

$x^3$ means $x \cdot x \cdot x$

Use your **calculator** to write these exponential numbers as a **decimal** and as a **fraction**.

<table>
<thead>
<tr>
<th></th>
<th>a) 10^{-1}</th>
<th>b) 10^{0}</th>
<th>c) 5^{-1}</th>
<th>d) 5^{0}</th>
</tr>
</thead>
<tbody>
<tr>
<td>e) 2^{-1}</td>
<td>f) 2^{0}</td>
<td>g) 4^{-1}</td>
<td>h) 4^{0}</td>
<td></td>
</tr>
<tr>
<td>i) 3^{-1}</td>
<td>j) 3^{0}</td>
<td>k) 2^{-2}</td>
<td>l) 4^{-2}</td>
<td></td>
</tr>
<tr>
<td>m) 10^{-2}</td>
<td>n) 3^{-2}</td>
<td>o) 5^{-2}</td>
<td>p) 10^{-3}</td>
<td></td>
</tr>
</tbody>
</table>

**e)** What effect does a negative sign have when it appears in an exponent? Was this what you expected?

**f)** What effect does zero have when it appears as an exponent?

Write the rule for what happens to numbers whose exponents are 0 or a negative in your own words.
#4

**BASE, EXPONENT, AND VALUE**

In the expression $2^5$, 2 is the **base**, 5 is the **exponent**, and the **value** is 32.

- $2^5$ means $2 \cdot 2 \cdot 2 \cdot 2 = 32$
- $x^3$ means $x \cdot x \cdot x$

Use exponents to write each of the following expressions as simply as possible. Look for patterns as you do this with your study team. Write out the variables to show the meaning whenever necessary.

Knowing that $\frac{y}{y} = 1$

$$y^3 \div y = \frac{y \cdot y \cdot y}{y} = y^2$$

Write these out the long way, like the example.

<table>
<thead>
<tr>
<th>a) $x^2 \div x$</th>
<th>h) $x^3 \div x^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) $y^2 \div y^5$</td>
<td>i) $x^3 \div x^2$</td>
</tr>
</tbody>
</table>

Write these out using the pattern or shortcut that you found.

<table>
<thead>
<tr>
<th>c) $(x^2) \div (x^5)$</th>
<th>j) $x^3 \div x^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>d) $x^7 \div x^5$</td>
<td>k) $m^{13} \div m^{14}$</td>
</tr>
<tr>
<td>e) $y^8 \div y^6$</td>
<td>l) $x^{32} \div x^{59}$</td>
</tr>
<tr>
<td>f) $y^7 \div y^4$</td>
<td>m) $x^{31} \div x^{29}$</td>
</tr>
<tr>
<td>g) $\frac{x^3}{x^1}$</td>
<td>n) $\frac{x^3}{x^3}$</td>
</tr>
</tbody>
</table>

Write the rule for dividing exponential numbers in your own words.
FORTY HOLES OF GOLF

The Hookenslice Corporation is having its annual charity fundraising event. In order to encourage donors to attend, Hookenslice organizes a fun game called “Forty Holes of Golf” and gives away prizes.

Each team plays forty holes of golf. There is a prize for the team that is consistently closest to the hole. Your teacher has set up a “hole.” Your team will “swing” forty pennies toward the “hole.” You will then represent your data on a graph and with numerical statistics. Analyzing the statistics will help you decide which team was the most consistently close to the hole.

Your Task:

• Your teacher will give you ten pennies. Have one team member stand 200 cm from the “hole.” That team member will toss all ten pennies. No “do-overs” and no practice shots are allowed. Then record the distance from the center of each penny to the “hole” (to the nearest centimeter), even if the penny rolled far away.

• Repeat with different team members until 40 pennies have been tossed. Do not take turns tossing pennies—each team member should toss all their pennies, one at a time. Then the next team member can take their turn.

• When directed by the teacher, return to the classroom. Decide how you want to represent your data on your poster: dot plot, boxplot, circle graph (“pie chart”), scatterplot, histogram, or bar graph. Create a poster. Leave room for the task below.

• Decide the five most important facts you wish to report about your team’s golf shots and add them to your poster.

• Record your team’s data in a safe place. You will need it in the next lesson.

• Your teacher will direct you on how to compare your team’s results with the other teams. Which team was most consistently close to the hole?
9-19. **Additional Challenge:** Use what you know about triangles and angle relationships to find the missing angles in the triangles below.
BP-52B  OVER THE PHONE ACTIVITY

Pretend you and your friend are talking to each other on the phone and you want your friend to draw the following diagram:

Your job is to write a clear set of directions using words only.

Use the space below to write your instructions in complete sentences. When you are finished you will exchange instructions with another student. DO NOT SHOW THE PICTURE TO THE OTHER STUDENT!

Over the phone directions: (Tear off and give these directions to another student.)

BP-52A  OVER THE PHONE ACTIVITY

Pretend you and your friend are talking to each other on the phone and you want your friend to draw the following diagram:

Your job is to write a clear set of directions using words only.

Use the space below to write your instructions in complete sentences. When you are finished you will exchange instructions with another student. DO NOT SHOW THE PICTURE TO THE OTHER STUDENT!

Over the phone directions: (Tear off and give these instructions to another student.)
TREASURE HUNT

Today your teacher will give you several descriptive clues about different relations. (This information is also available online at www.cpm.org.) For each clue, work with your team (or a partner) to find all the possible matches among the relations posted around the classroom or provided on the resource page. Remember that more than one relation may match each clue. Once you have decided which relation(s) match a given clue, defend your decision to your teacher and receive the next clue. Be sure to record your matches on paper.

Your goal is to find the match (or more than one match) for each of eight clues. Once you and your team (or partner) have finished, only one relation will be left unmatched. That relation is the
Clue Cards

Treasure-Hunt Clues A
Find one (or more) relation(s) that:
A2. Has a domain of all numbers except $x = 3$.
A3. Has $x$-intercepts $(2, 0)$ and $(8, 0)$.
A4. Is not a function.
A5. Has a range of all numbers less than or equal to 3.
A6. Is a linear relation.
A7. Has a range of all numbers.
A8. Has a $y$-intercept at $(0, -1)$.

Treasure-Hunt Clues B
Find one (or more) relation(s) that:
B1. Is not a function.
B3. Has a range of all numbers less than or equal to 3.
B4. Has a range of all numbers.
B5. Has a $y$-intercept at $(0, -1)$.
B6. Has a domain of all numbers except $x = 3$.
B7. Has $x$-intercepts $(2, 0)$ and $(8, 0)$.
B8. Is a linear relation.

Treasure-Hunt Clues C
Find one (or more) relation(s) that:
C1. Is a linear relation.
C2. Has a range of all numbers.
C3. Is not a function.
C4. Has $x$-intercepts $(2, 0)$ and $(8, 0)$.
C5. Has a domain of all numbers except $x = 3$.
C6. Has a $y$-intercept at $(0, -1)$.
C7. Has a range of all numbers less than or equal to 3.
C8. Has no symmetry.

Treasure-Hunt Clues D
Find one (or more) relation(s) that:
D1. Has a range of all numbers less than or equal to 3.
D2. Has $x$-intercepts $(2, 0)$ and $(8, 0)$.
D3. Has a domain of all numbers except $x = 3$.
D4. Is a linear relation.
D5. Has no symmetry.
D6. Has a range of all numbers.
D7. Has a $y$-intercept at $(0, -1)$.
D8. Is not a function.
# Treasure Hunt Answer Key

<table>
<thead>
<tr>
<th>Team A Solutions</th>
<th>Team B Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1. 5, 8</td>
<td>B1. 5, 6</td>
</tr>
<tr>
<td>A2. 4</td>
<td>B2. 5, 8</td>
</tr>
<tr>
<td>A3. 2, 3, 5, 7</td>
<td>B3. 2, 8</td>
</tr>
<tr>
<td>A4. 5, 6</td>
<td>B4. 6, 10</td>
</tr>
<tr>
<td>A5. 2, 8</td>
<td>B5. 4, 5, 10</td>
</tr>
<tr>
<td>A6. 10, 1</td>
<td>B6. 4</td>
</tr>
<tr>
<td>A7. 6, 10</td>
<td>B7. 2, 3, 5, 7</td>
</tr>
<tr>
<td>A8. 4, 5, 10</td>
<td>B8. 10, 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Team C Solutions</th>
<th>Team D Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1. 10, 1</td>
<td>D1. 2, 8</td>
</tr>
<tr>
<td>C2. 6, 10</td>
<td>D2. 2, 3, 5, 7</td>
</tr>
<tr>
<td>C3. 5, 6</td>
<td>D3. 4</td>
</tr>
<tr>
<td>C4. 2, 3, 5, 7</td>
<td>D4. 10, 1</td>
</tr>
<tr>
<td>C5. 4</td>
<td>D5. 5, 8</td>
</tr>
<tr>
<td>C6. 4, 5, 10</td>
<td>D6. 6, 10</td>
</tr>
<tr>
<td>C7. 2, 8</td>
<td>D7. 4, 5, 10</td>
</tr>
<tr>
<td>C8. 5, 8</td>
<td>D8. 5, 6</td>
</tr>
</tbody>
</table>
Treasure Hunt

Relation #1:

Relation #2:

Relation #3:

Relation #4:

\[ y = \frac{3}{x-3} \]

Relation #5:

\[
\begin{array}{c|c}
 x & f(x) \\
\hline
\frac{1}{2} & 17 \\
-1 & 7 \\
2 & 0 \\
0 & -1 \\
5 & -6 \\
8 & 0 \\
-1 & 2 \\
3 & 6 \\
7 & 0.5 \\
\end{array}
\]

Relation #6:
**Treasure Hunt**

**Relation #7:**

\[ f(x) \text{ is a quadratic function.} \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>16</td>
<td>7</td>
<td>0</td>
<td>−5</td>
<td>−8</td>
<td>−9</td>
<td>−8</td>
<td>−5</td>
</tr>
</tbody>
</table>

**Relation #8:**

![Graph of a quadratic function]

**Relation #9:**

\[ y = |x| \]

**Relation #10:**

\[ y = \frac{1}{2} x - 1 \]
<table>
<thead>
<tr>
<th>RED</th>
<th>RED</th>
<th>( y = 2x + 1 )</th>
<th><img src="image" alt="Graph" /></th>
<th>Describe what you see:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Multiply your number by two and add one.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BLUE</th>
<th>BLUE</th>
<th>( y = -2x )</th>
<th><img src="image" alt="Graph" /></th>
<th>Describe what you see:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Multiply your number by negative two.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>YELLOW</th>
<th>YELLOW</th>
<th>( y = x + 4 )</th>
<th><img src="image" alt="Graph" /></th>
<th>Describe what you see:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Add four to your number.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GREEN</th>
<th>GREEN</th>
<th>( y = -x + 4 )</th>
<th><img src="image" alt="Graph" /></th>
<th>Describe what you see:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Change the sign of your number and add four.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ORANGE</th>
<th>ORANGE</th>
<th>( y = x^2 )</th>
<th><img src="image" alt="Graph" /></th>
<th>Describe what you see:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Square your number. That is, multiply your number by itself.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
THE BIG RACE – FINALS

Today is the final event of “The Big Race”! Your teacher will give you each a card that describes how you travel in the race. You and your study team will compete against the heat 1 and 2 winners, Leslie and Elizabeth, at today’s rally in the gym. Unfortunately, Mark, the winner of heat 3, is absent from school and will not be participating against you.

Your Task: As a team, do the following:

• Draw a graph (on graph paper) showing all of the racers’ progress over time. Identify the independent and dependent variables.
• Write an equation for each participant.
• Figure out who will win the race!

Rules:

• Your study team must work cooperatively to solve the problems. No team member has enough information to solve the puzzle alone!
• Each member of the team will select rider A, B, C, or D. You may not show your card to your team. You may only communicate the information contained on the card.
• Assume that each racer travels at a constant rate throughout the race.
• Elizabeth’s and Leslie’s cards will be shared by the entire team.

Use your results from “The Big Race – Finals” to answer the following questions. You may answer the questions in any order, but be sure to justify each response.

a. Who won the finals of The Big Race? How do you know? Who came in last place? When will she finish the race?

b. How fast was Rider D traveling? How long did it take for him/her to finish the race? How fast was Elizabeth traveling?

c. At one point in the race, four different participants were the same distance from the starting line. Who were they and when did this happen?

d. If Rider D was not in the race, and the Race was only 15 meters long, who would win? How do you know?
The beautiful young princess of **Polygonia** is very sad. A mean ogre has locked her into a tower of a castle. She could escape through the window, but it is 50 feet above the ground, a long distance to jump! A moat full of alligators surrounds the tower. Naturally, Prince Charming wants to rescue her.

The prince has some rope. His plan is to use an arrow to shoot one end of the rope up to her window. The princess can then slide down the rope to the other side of the moat, and off they will ride into the sunset.

Help the prince save the princess by answering the questions below.

a. The prince knows that the closer he is to the tower, the less rope he will need. However, he is not sure how wide the moat is around the castle.

   In a book about the castle, the prince found a diagram of the tower and moat. Using his ruler, he found that the window in the diagram is 7.5 inches above the moat, while the farthest edge of the moat is 1.5 inches from the base of the tower. How close to the tower can the prince get? Draw diagrams and show your work.

b. Before he shoots the arrow, the prince wants to make sure he has enough rope. If he needs an extra 2 feet of rope for tying it to the window frame and holding it on the ground, how long does his rope need to be? Explain.

c. The princess thinks she might fall off the rope if the slope is steeper than $\frac{5}{4}$.

   The prince plans to attach the rope to the ground at the edge of the moat, and the princess will attach the rope to the window frame 50 feet above the ground.

   i. According to the prince’s plan, what would the slope of the rope be? Would it be too steep? Justify your answer.

   ii. Should the princess worry about the prince’s plan? How steep would the prince’s rope be? Find the angle the rope would make with the ground. Use a protractor to measure the angle.

d. If one end of the rope is attached to the window frame and the prince holds his end of the rope 2 feet above the ground, where would the prince need to stand so that the slope of the rope is only $\frac{5}{4}$? If he has a rope that is 62 feet long, will his rope be long enough? Justify your conclusion. Draw a picture and label it with all your measurements.

e. What if the prince holds his end of the rope 5 feet above the ground? Can he stand in some location so that the rope is not steeper than $\frac{5}{4}$? (The rope is still 62 feet long.) Justify your conclusion.
CL 1-83. Use the Order of Operations to simplify the following expressions.

a. $5 - 2 \cdot 3^2$

b. $(-2)^2$

c. $18 + 3 \cdot 6$

d. $-2^2$

e. $(5 - 3)(5 + 3)$

f. $24 \cdot \frac{1}{4} \div -2$

g. Why are your answers for parts (b) and (d) different?

CL 1-85. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.

a. 
-xy 
9
\-9

b. 
-36
0

C. 
-20
4

D. 
\frac{3}{2}

CL 1-86. Graph and fully describe the function $y = 2\sqrt{x - 1} + 3$.

CL 1-87. Solve each equation. Check your solution.

a. $3x - 1 = 4x + 8 - x$

b. $-10 + 5x = 7x - 4$

c. $28 - 6x + 4 = 30 - 3x$

d. $4x - 1 = 9x - 1 - 5x$

CL 1-88. Find $f(4)$ for each function below.

a. $f(x) = -|x - 7| + 3$

b. $f(x) = \frac{\sqrt{x+12}}{4}$

c. $f(x) = 2 - 3\sqrt{x + 23}$

CL 1-89. Evaluate each expression.

a. $2 + |3 - 4|$

b. $11 - |6| + 15$

c. $-19 + \sqrt[3]{-8}$

d. $-11 - \sqrt{16}$

CL 1-90. Use the function machine shown at right to answer the following questions.

a. If the input is $-8$, what is the output?

b. If the output was 21, what was the input?
DOT PATTERN

1-16 Copy the dot pattern below onto graph paper.

![Dot Pattern Images]

- Figure 1
- Figure 2
- Figure 3

a. What should the 4th and 5th figures look like? Draw them on your paper.

b. How can you describe the way the pattern is growing? Can you find more than one way?

c. How many dots would be in the 10th figure of the pattern? What would it look like? Draw it.

d. How many dots would be in the 30th figure? How can you describe the figure without drawing the entire thing? Can you describe it with words, numbers, or a simple diagram? Be ready to explain your ideas to the class.

Make sure that each person has a copy of the work and that you have:

- Clear drawings of the 4th and 5th figures of the pattern. Use color to help you show how you see the pattern.

- An explanation of the different ways that you see the pattern you found. Find ways to help your classmates understand how you saw the pattern.

- Your prediction for the 30th figure with a clear explanation.

1-17 Work with your team to find a way to describe any figure in the pattern. In other words, if you knew a figure number, how could you decide what the figure looks like even if you cannot draw it? Be ready to share your ideas with the class.
Human Bar Graph: In the front of the classroom, have signs that say “Quadrilateral,” “Triangle,” and “Other Polygons.” Students will use these shapes to construct a human bar graph. Ask students to line up in the front of the sign that has their polygon name. Have them form a line, one person in back of another so it makes a bar graph.

Human Linear Model: Have the students reform themselves into a line where all the quadrilaterals are at one end and any other polygon at the other end. As students look around, ask students “What portion of the people have a quadrilateral?” and “What portion do not have a quadrilateral?” However, it still may be hard to see the entire class. This motivates the need to change the representation.

Human Circle Graphs: Propose that students curve the line to form a circle in the classroom so they can all see everyone’s shapes. It is not necessary to clear the center of the classroom, but students may need to move desks around a bit. You should be in the center of the circle. To visually separate the class into the two portions being discussed, give one of the students on each of the borders of the two teams one end of the yarn to hold while you hold the other end at the center. This is the beginning of the circle graph, as shown in the following diagram (birds-eye view).

Once the circle is formed, ask the questions again, “About what portion of the people have a quadrilateral?” and “What portion do not have a quadrilateral?” Expect estimates in both fraction and percent form. If students are not lined up evenly, ask students to consider how many individuals represent parallelograms. Ask whether the size of the section of the circle roughly matches with the portion of students in the class who hold parallelograms, and to adjust their spacing if it does not. Point out how visibly the circle shows each part relative to the whole.

Introduce the concept of a central angle, located at the center of a circle, and use this opportunity for students to use basic deduction to estimate the size of central angles. Ask, “If this entire circular angle at the center measures 360°, then what angle should be formed by the strings at the center of the circle? Why?” Due to the inexact nature of this human circle graph, students may need to move slightly to more reasonably reflect the estimated angles. It may also help to ask them questions about whether the angle they are describing should be more or less than 90° based on whether it is more or less than one-fourth of the circle.

To continue to make additional sectors of the circle graph ask the students in the quadrilateral team to raise their hand if they are holding a parallelogram. Ask, “What portion (i.e., fraction or percent) of the students in the circle holds parallelograms?” Students may recognize the need to rearrange themselves along the circle to group parallelograms together. Again, provide yarn to the students who now separate the parallelograms from the other quadrilaterals. You should now have a human circle graph partitioned into three sections (parallelograms, other quadrilaterals, and non-quadrilaterals).

Discuss the measure of the central angles that were created. Ask questions such as, “Are any of these portions 60%? Does that mean this is a 60° angle? Why or why not?” If time allows, provide students the opportunity to suggest other categorizations of the shapes, such as those that are equilateral versus those that are not. For each re-categorization, students will need to re-sort themselves in the circle.
## Shape Cards

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="square.png" alt="Square" /></td>
<td><img src="rectangle.png" alt="Rectangle" /></td>
</tr>
<tr>
<td><img src="rhombus.png" alt="Rhombus" /></td>
<td><img src="hexagon.png" alt="Hexagon" /></td>
</tr>
<tr>
<td><img src="parallelogram.png" alt="Parallelogram" /></td>
<td><img src="trapezoid.png" alt="Trapezoid" /></td>
</tr>
</tbody>
</table>
Shape Cards

[Diagram of various shapes]
Shape Cards

- Triangle
- Triangle
- Triangle
- Triangle
- Pentagon
- Quadrilateral
Win-A-Row Game Boards

\[ \begin{array}{cccccccc} 
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
-1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 \\
\end{array} \]

\[ \begin{array}{cccccccc} 
\text{sum} \\
\hline \\
\hline \\
\end{array} \]
Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.
   • Find meaning in problems
   • Look for entry points
   • Analyze, conjecture and plan solution pathways
   • Monitor and adjust
   • Verify answers
   • Ask themselves the question: “Does this make sense?”

2. Reason abstractly and quantitatively.
   • Make sense of quantities and their relationships in problems
   • Learn to contextualize and decontextualize
   • Create coherent representations of problems

3. Construct viable arguments and critique the reasoning of others.
   • Understand and use information to construct arguments
   • Make and explore the truth of conjectures
   • Recognize and use counterexamples
   • Justify conclusions and respond to arguments of others

4. Model with mathematics.
   • Apply mathematics to problems in everyday life
   • Make assumptions and approximations to simplify a complicated situation
   • Identify quantities in a practical situation
   • Interpret results in the context of the situation and reflect on whether the results make sense

5. Use appropriate tools strategically.
   • Consider the available tools when solving problems
   • Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)
   • Make sound decisions of which of these tools might be helpful

6. Attend to precision.
   • Communicate precisely to others
   • Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes
   • Calculate accurately and efficiently

7. Look for and make use of structure.
   • Discern patterns and structures
   • Can step back for an overview and shift perspective
   • See complicated things as single objects or as being composed of several objects

8. Look for and express regularity in repeated reasoning.
   • Notice if calculations are repeated and look both for general methods and shortcuts
   • In solving problems, maintain oversight of the process while attending to detail
   • Evaluate the reasonableness of their immediate results