Effective Differentiation of Both Ends of the Spectrum in 6-12 Math

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Good Morning

- **Welcome to NESA!**
- **Welcome to two days about differentiation – that is, better meeting the needs of all learners – in secondary mathematics classrooms.**
- **Take a deep breath, relax, introduce yourself to your neighbors, and let's dig in.**

Workshop Goal:

Amid the clarion call from others and within ourselves to "differentiate, differentiate, differentiate," we know we need to do a better job meeting the mathematical needs of ALL students. It's so convenient to focus on the middle and take shortcuts with the strongest and weakest 20%.

Accordingly, these two days will focus on practical strategies, with lot of examples, for balancing the needs of struggling and high flyer students so that you leave with a deeper understanding of how better to meet the needs of diverse learners in our secondary mathematics classroom.

A 2-day Approximate Agenda

Day 1 morning: Overview, examples and discussion of more effective mathematics teaching and learning with a focus on the strongest and weakest 20%

Day 1 before and after lunch: Strategies that maximize access to and interest in the math for ALL

Day 1 afternoon: Focusing on our struggling students

Day 2 morning: Focusing on our high flyers

Day 2 afternoon: Resources and practice putting it all together

End of Day 2: Next steps and closing thoughts

Me – www.steveleinwand.com

- **45 years**
- **3 jobs**
- **About 200,000 air mile per year (about 400 hours or 50 8-hour work days for reading, sleeping and catching up)**
- **Representations because the members of every class learn in different ways**
- **Most PD…**
- **Validate outliers and sprinkle empowerment dust**
- **My plea for breaking the cycle by reducing our professional isolation.**

Ready???

Let’s do some math.

Straight from the textbook

Sarah has picked 2605 apples. She has 91 boxes. How many apples will Sarah put in each box if each box holds the same number of apples?

UGH! Brain-numbing! But so typical.

Pluck out the numbers. Convert “in each” to divide.

Divide and done.

Now do 7 more just like it with little additional learning.

Adapting what the text bestows by turning practice into opportunities for learning with a focus on our questions.

Sarah has 91 empty boxes.

- What can you infer about Sarah?

She had 2605 apples to pack into the boxes.

- Now what can you infer about Sarah?
- What do you think the question is?
- So what does the 91 tell us, what about the 2605?
- About how many apples do you think would be in each box? More than 100? Less than 100? Convince us
- Can you draw a picture?
- Can you create a number sentence?
- Do you multiply or divide? Why?
- So about how many apples would be in each box if…

Sarah and her silly apples

What did the shift from one large dump to scaffolded questions do to support ALL students?

Let’s Try It

Grade 8/9

Josh and his family went to the carnival. It costs $5 to park the car plus $1.50 for each ride.

Josh’s family spent a total of $38. How many rides did they go on?

Write an equation and show your work.

UGH! Just same old pretty boring, narrow math that elicits either I don’t understand or I don’t care.

Alternatively: Turn and tell your partner what we can do with this to better serve all students.

Your turn

Welcome to the Carnival
Parking $5/car
Rides $1.50 each

What do you notice?
??
??
What do you wonder?
??
??
Let’s play with what Robert wondered…
Great, now let’s play with was Sierra wondered…

Moving from tasks to lessons

Grade 6 SA Harlem Central Tues Dec 8, 2015

- **Lesson 6 in the Expressions unit (6.EE standards)**
- **Ally and Mabubar co-teaching**
- **19 Scholars**
- **Driven by a number strings mini-lesson, a Math Workshop task and an exit ticket**
- **“Our goal for today is to “identify, create and understand equivalent expressions.”**
- **“Zayasia, can you please repeat our learning goal?”**
- **“Let’s begin with out number strings.”**

Number strings for today's Mini Lesson

Are they equivalent? How do you know?

1. **$4(8) = 4(3 + 5)$**

2. **$4(8) = 4(a + 5)$**

3. **$4(8) = 4(3 + b)$**

4. **$3x + 3y = 3(x + y)$**

Let's summarize: For each: Always, sometimes, never equivalent?

Math Workshop Task

Jan normally rides her bike to and from work.

Her normal route is 18 miles from home to work.

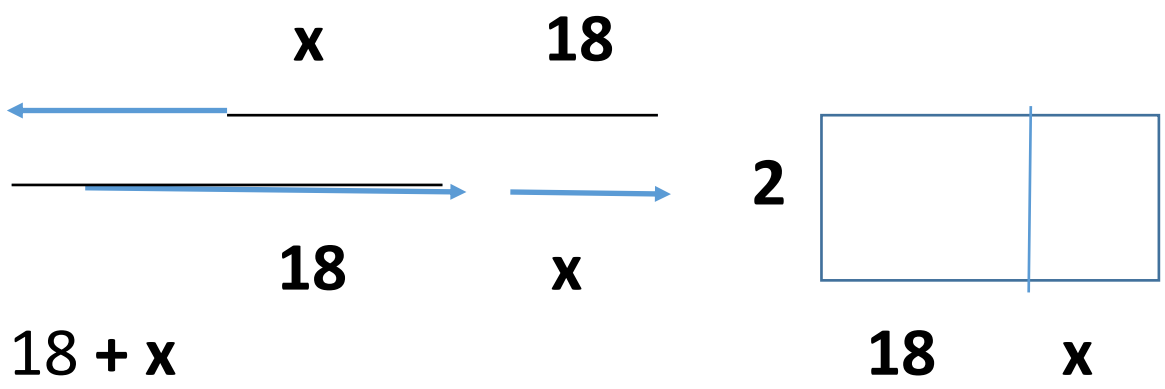
One day she goes to a coffee shop on her way to work and on her way home.

This adds x miles to her trip each way.

("What do you notice?" "What's the question?")

Great: Write and show the distance Jan travels using a diagram or picture and two different, but equivalent, expressions.

Resulting in:



- 18 + x + 18 **+ x**
- **2(18 + x)**
- **2(x + 18)**
- **x + x + 18 + 18**
- **36 + 2x**

“Is everyone correct? Turn and tell your partner why?”
“What do the numbers and variables represent?”
“Which expression is simplest or easiest to use? Why?”

Lesson 6 Exit Ticket

Which of the following represent equivalent expressions?

Explain or show your process of determining which expressions ARE equivalent.

Select all that apply:

a. $x + x + x + x = 4x$

b. $15y + 5x = 3(5y + x)$

c. $6(2 + x) = 12 + 6$

d. $3(x + y) = 3x + y$

Turn and tell you neighbor why you think this 3-slide lesson was effective?

What did Aly and Mabubar do to meet diverse needs of students?

Dan Meyer’s Taco Cart

http://threeacts.mrmeyer.com/tacocart/

Taco Cart

- **How did we give ALL students access to this complex task?**
- **What else could we/you have done to provide more/better access?**
- **How did we provide opportunities for our high flyers?**

Elements of Quality

- Clarity of goals (not Lesson 4.5 or pages 214-217)
- Context (not naked)
- Rich tasks (not exercises)
- Focused intentional questions (not punting)
- Opportunities for discourse (not just telling)
- Gradual release (not just a dumping)
- Multiple representations (not one way)
- Alternative approaches (not one way)
- Explanations and justifications (not just answers)
- Common errors and misconceptions (not just right correct approaches)
- Sense-making by students (not lecture)
- Evidence (not I taught it and let the chips….)

We know what it takes

What we strive for: LEARNING SUCCESS

The enabling conditions: ENGAGEMENT
PARTICIPATION

What it takes: TASKS QUESTIONS DISCOURSE
MISTAKES PRODUCTIVE STRUGGLE

Enhanced by: REPRESENTATIONS
ALTERNATIVES
ESTIMATES

All guided by: A CLARITY OF LEARNING GOALS

One by one, just consider how each of these 8 MTPs live in this lesson?

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

*** MTPs = Mathematical Teaching Practices from NCTM's Principles to Actions**

So our big ideas for these two days:

- **More open-ended tasks**
- **Low floors and high ceilings**
- **Their learning is driven by our questions**
- **The heart of differentiation is:**
  - ✓ **Multiple representations**
  - ✓ **Alternative approaches**

Reading break:

Turn to Thoughts on Making Inquiry-based, Conceptually-driven, Sense-making Mathematics the Enacted Norm in Every Mathematics Class Every Day.

5 minutes to read

3 minutes to reflect on what resonates and what is troubling

3 minutes to share your reflections with a partner

4 minutes to share out insights and questions

Pause and Reflect

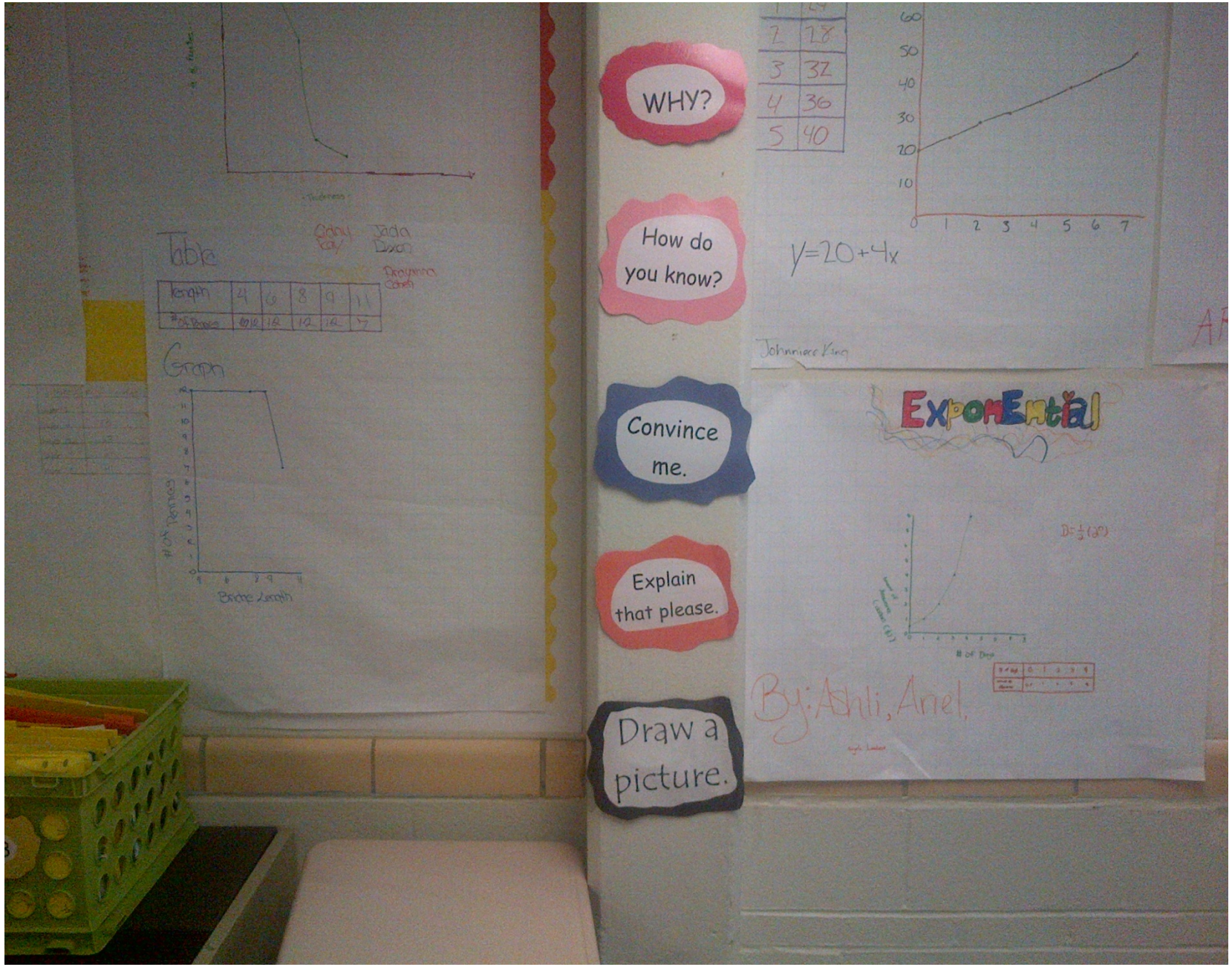
- **How has what you already do been reinforced?**
- **What initial aha's have you had?**
- **What questions have arisen as a result of our introductory tasks?**

Strategies for supporting both ends of the spectrum

- Questions that go beyond just the answer
- Give them a reason to care - contexts
- On-going cumulative review
- Gradual reveal
- Freyer model

Our questions….

- **Determine their learning**
- **Stimulate their thinking**
- **Encourage explanation and justification**


WHY?
How do you know?
Convince me.
Explain that please.
Draw a picture.
y=20+4x
Johnniece King
ExponEntial
By: Ashli, Ariel,
Table
Graph
Brine Length

Classroom discourse and representations

- **Why?**
- **How do you know?**
- **Can you explain that? (and who did it differently?)**
- **Convince us.**
- **How did you picture that? (and who did it differently?)**
- **What do you notice?**
- **What do you wonder?**
- **How are they the same?**
- **How are they different?**

Context

Contexts

- **Menus**
- **Recipes**
- **Price lists**
- **Picnics**
- **Pizza Orders**
- **Garden Designs**
- **Data Sets**

- **Add to this list.**

What’s the question? What’s the answer?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3

- **Will you make it?**
- **About how far will you get?**
- **How much gas needs to be in the tank to be sure you make it?**

You choose:

1.59) 10

vs.

You have $10. Big Macs cost $1.59

SO?

You choose….

- **The one right way to get the one right answer that no one cares about and isn't even asked on the state tests**

vs.

- **Where am I? (the McDonalds context)**
- **Ten? Convince me.**
- **About how many? How do you know?**
- **Exactly how many? How do you know?**
- **Oops – On sale for $1.29 and I have $20.**

You choose:

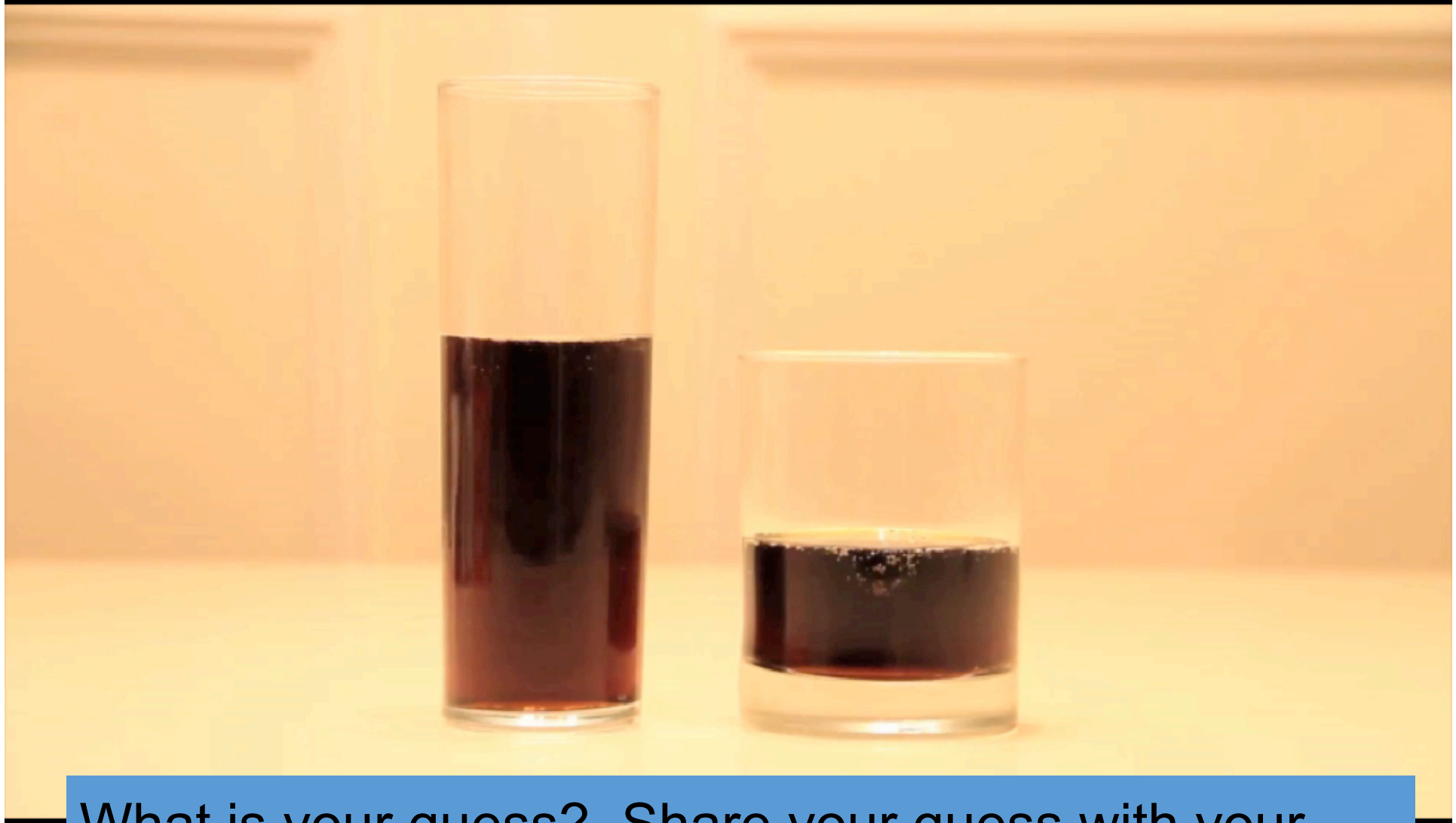
Given: $F = 4(S - 65) + 10$

Find F when S = 81.

Vs.

The speeding fine in Vermont is $4 for every mile per hour over the 65 mph limit plus $10 handling fee.

Tell your partner three things you see here.



What is your guess? Share your guess with your neighbor and justify your guess.

What information is important here?
How would you get it?

SOME QUESTIONS FOR YOU:

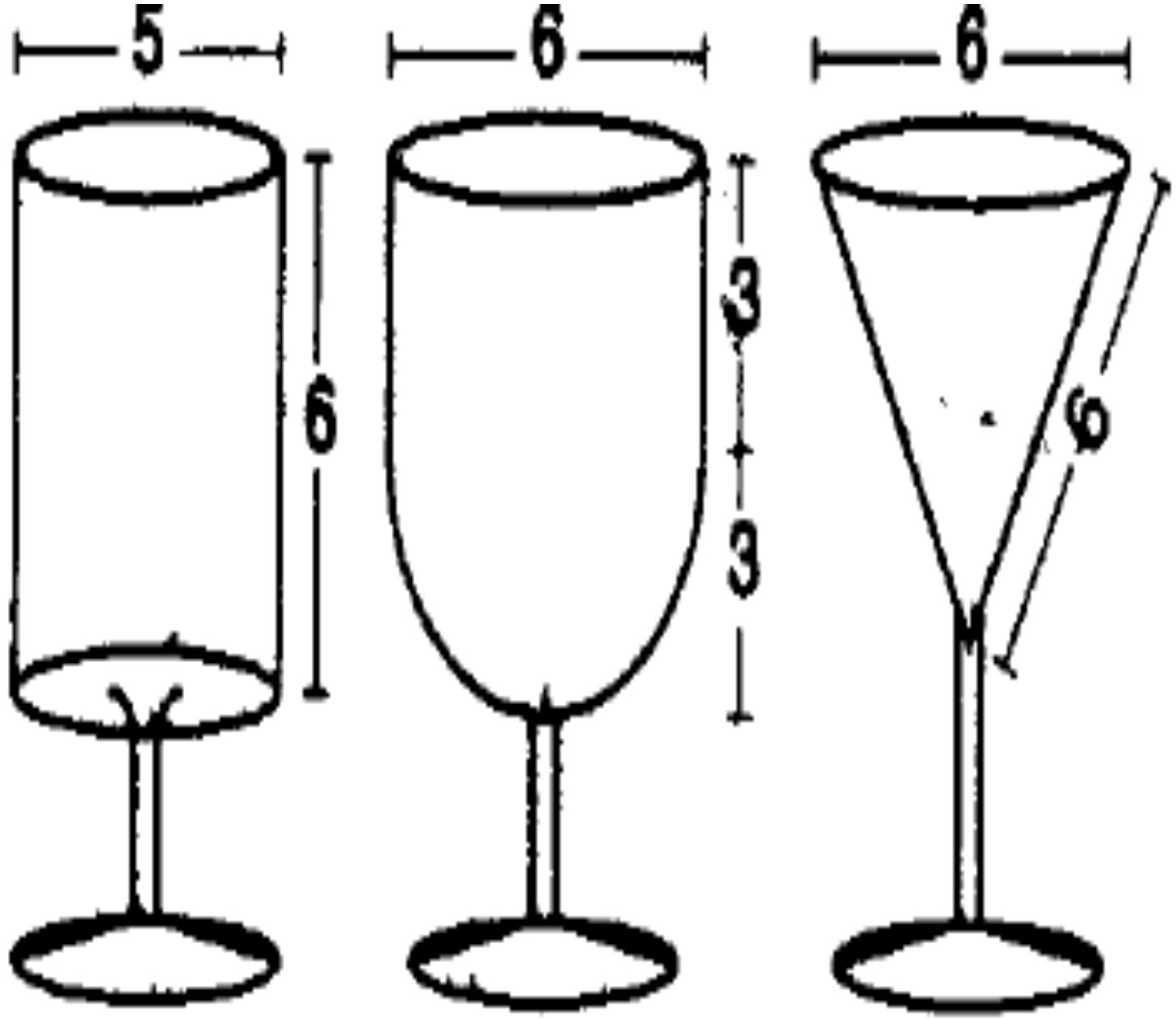
- **What mathematics?**
- **Why does this engage you?**
- **Why are worksheets still so prevalent?**
- **How do we bring these shifts to scale?**

A FORMATIVE ASSESSMENTFOR THEM:

- **Now draw two glasses with different diameters and show the heights of equal amounts of liquid. Explain your reasoning.**

And how do we enrich?

(we steal from Balanced Assessment and/or the Math Assessment Project)



1 2 3

So?

- **Order from smallest to largest and justify**
- **What is the height of Glass 3?**
- **What is the volume of each?**
- **If Glass 1 has volume V, express volume of Glasses 2 and 3 in terms of V**
- **When Glass 1 is ½ full, the height of the liquid is 3 cm. What are the heights of the liquid in Glasses 2 and 3 when they are ½ full?**

Cumulative Review

Almost no one masters something new after one or two lessons and one or two homework assignments. That is why one of the most effective strategies for fostering mastery and retention of critical skills is daily, cumulative review at the beginning of every lesson.

Number from 1 to 6

1. **What is 6 x 7?**
2. **What number is 1000 less than 18,294?**
3. **About how much is 32¢ and 29¢?**
4. **What is 1/10 of 450?**
5. **Draw a picture of 1 2/3**
6. **About how many square meters is this room? Closer to 100, 1000, 10,000 or 100,000**

Good morning Boys and Girls
Number from 1 to 5

1. **What is the value of tan (π/4)?**
2. **Sketch the graph of $(x-3)^2 + (y+2)^2 = 16$**
3. **What are the equations of the asymptotes of $f(x) = (x-3)/(x-2)$?**
4. **If $\log_2 x = -4$, what is the value of x?**
5. **About how much do I weight in kg?**

Gradual Reveal

Instead of bombarding students with the whole word problem, the entire graph or figure or table, use the power of PowerPoint to gradually release or reveal the problem, graph, figure, etc. using questions to probe understanding of prior and new content.

Why Bother?

- Opportunities to build interest
- Opportunities to scaffold
- Opportunities for inferential reasoning
- Opportunities to engage with low threshold questions
- Opportunities to introduce open-ended questions

Let’s Play

Using gradual release with:

- Word problems
- Tables
- Graphs
- Patterns
- Geometric figures

Graphs – a great task – but overwhelming

The pie chart represents the amount of money collected by various stalls at a funfair.

(a) What fraction of the total amount of money was collected by the games stalls?

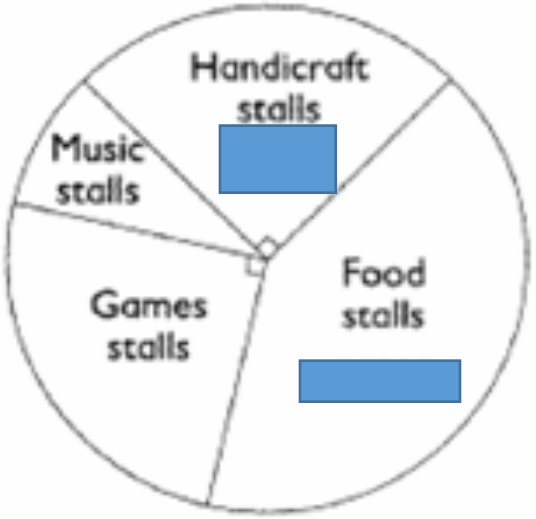
(b) What was the total amount of money collected by the various stalls?

(c) How much money was collected by the music stalls?

(d) What was the ratio of the money collected by the food stalls to the money collected by the handicraft stalls?

Graphs – a great task – gradually revealed

What do you notice?
What do you wonder?



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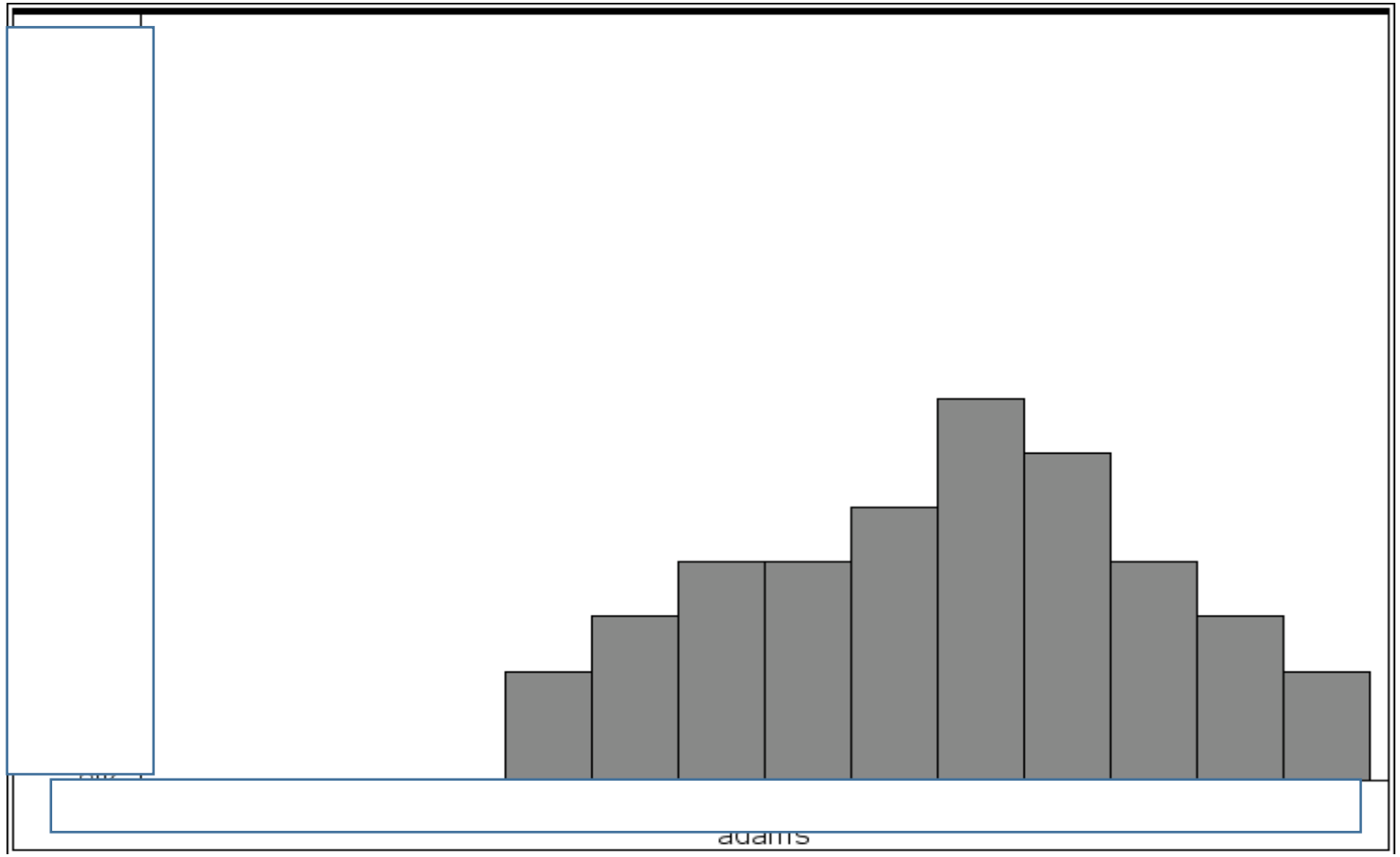
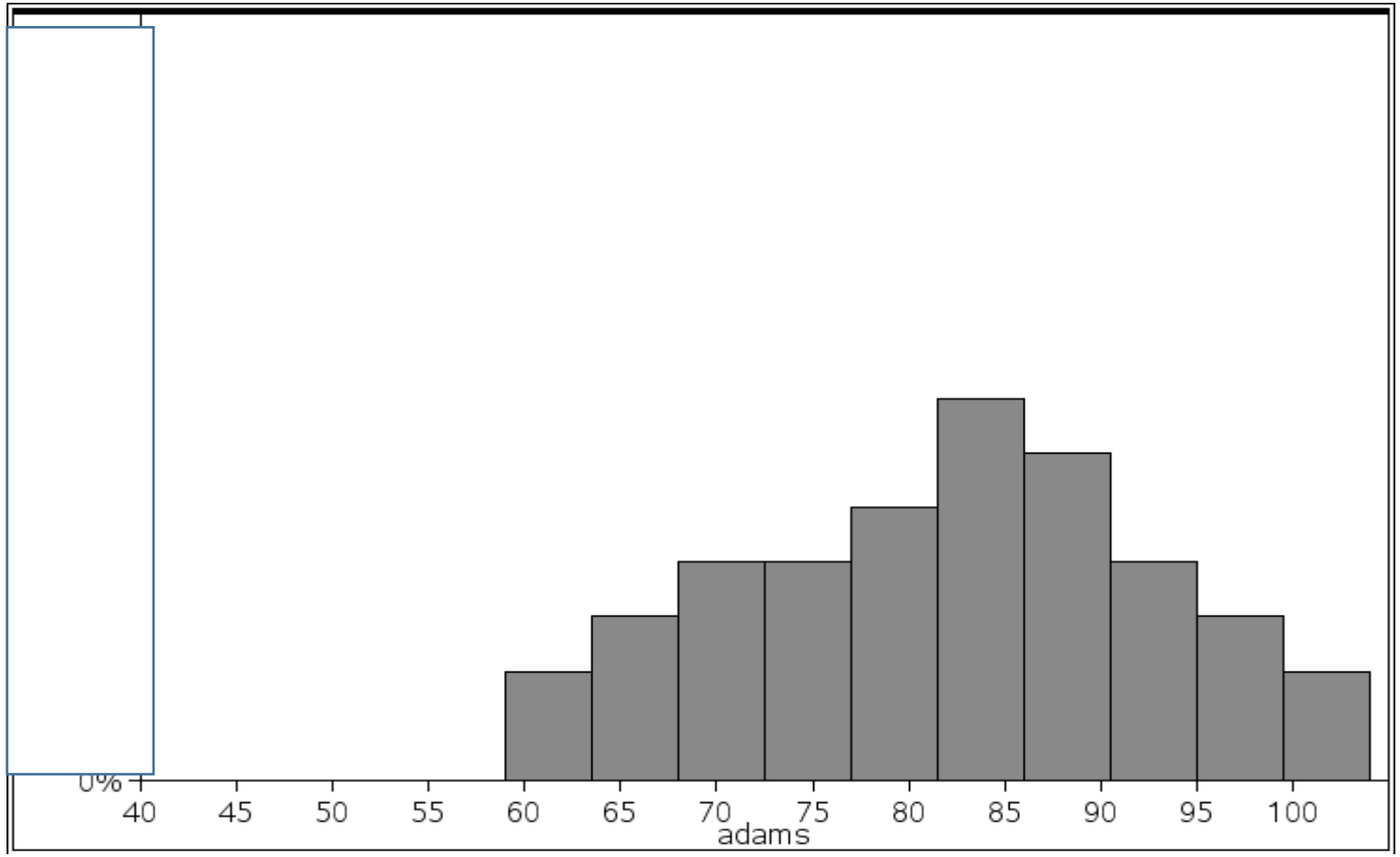
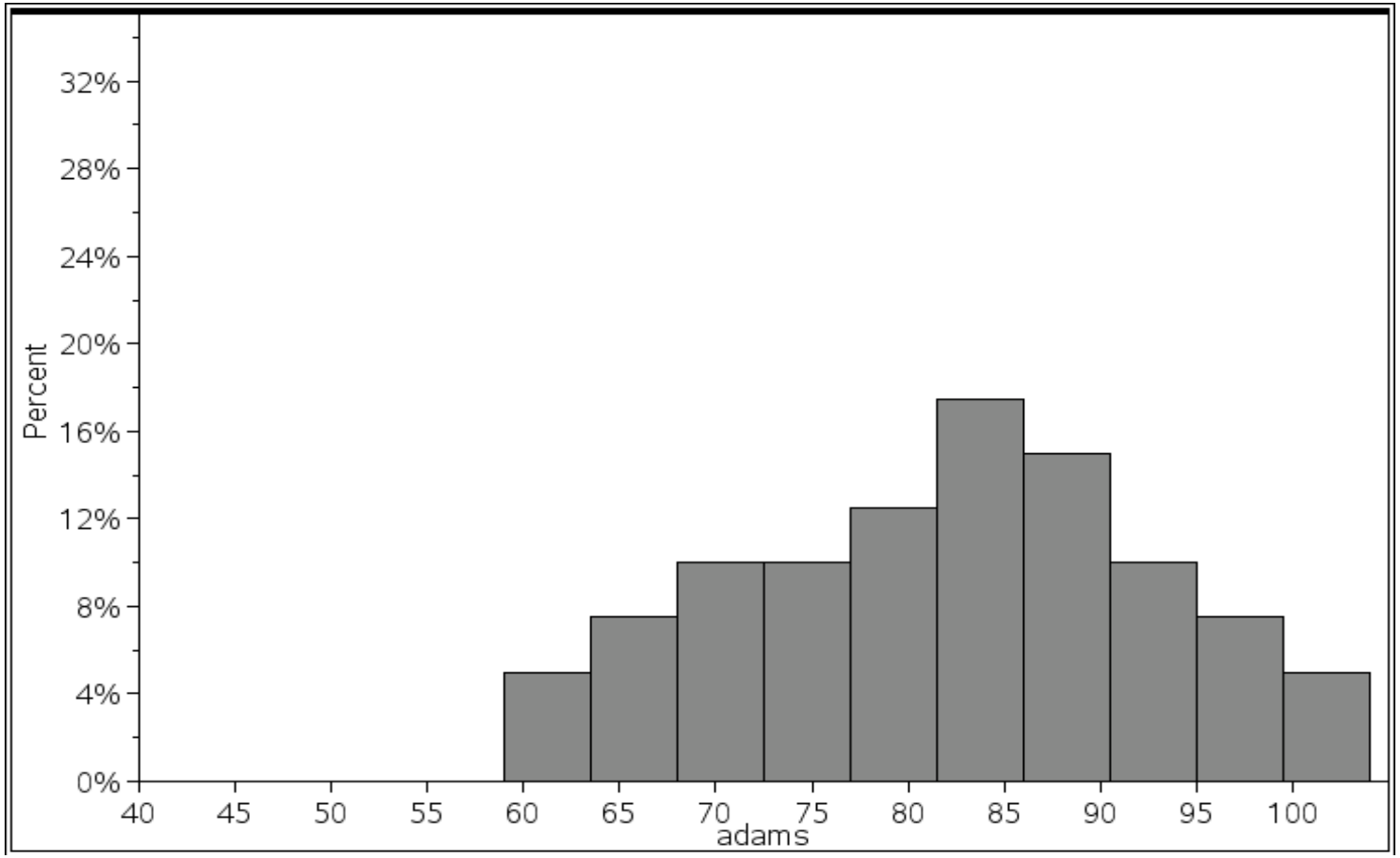
What’s the story?



What’s the story?



What's the title? What's the opening paragraph?



Geometric Figures

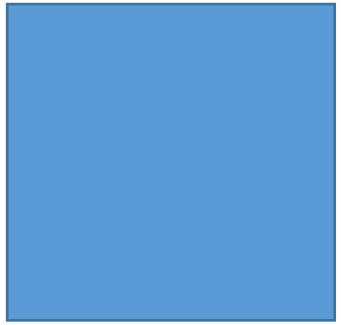
Name these figures.

A little better: How are these three shapes the same? How are they different?

Alternatively, with gradual reveal:

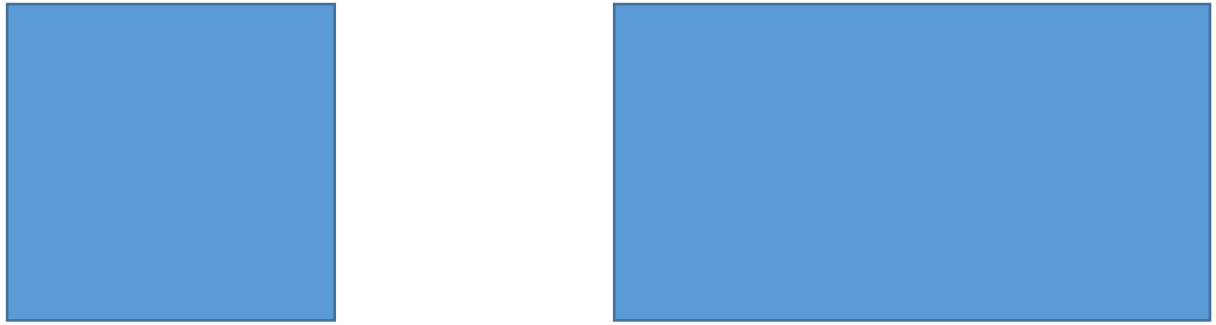
Geometric Figures

What do you notice?



Geometric Figures

Now what do you notice? How are they the same? Different?



Geometric Figures

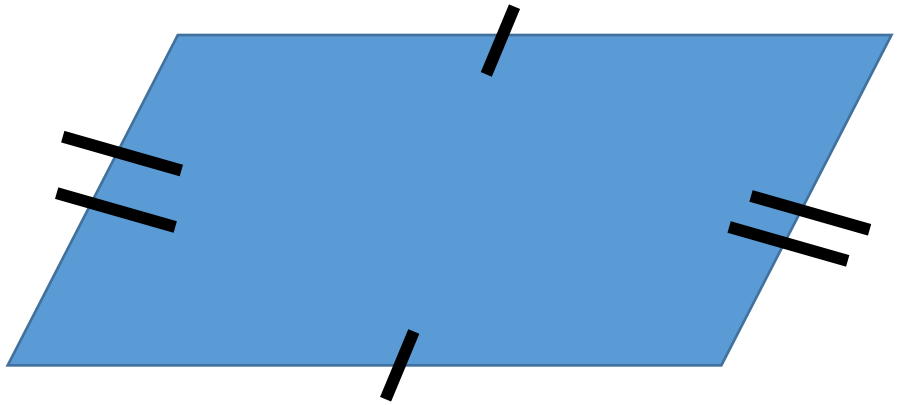
How are these three shapes the same?

How are these three shapes different?

What relationships between squares, rectangles and parallelograms can you identify?

Geometric Figures – OY!

What do you notice?



Convergent math. It's a parallelogram because……..

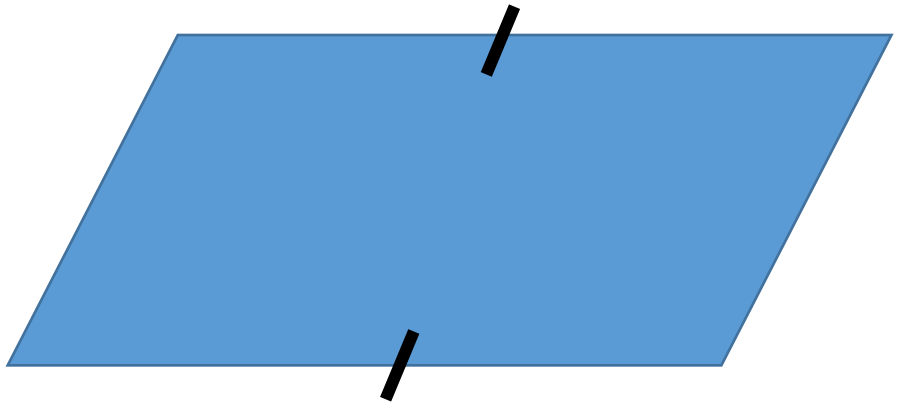
Geometric Figures – revealed gradually

What do you notice and know for sure?

What do you think might be true, but can't be sure? Why not?

Geometric Figures

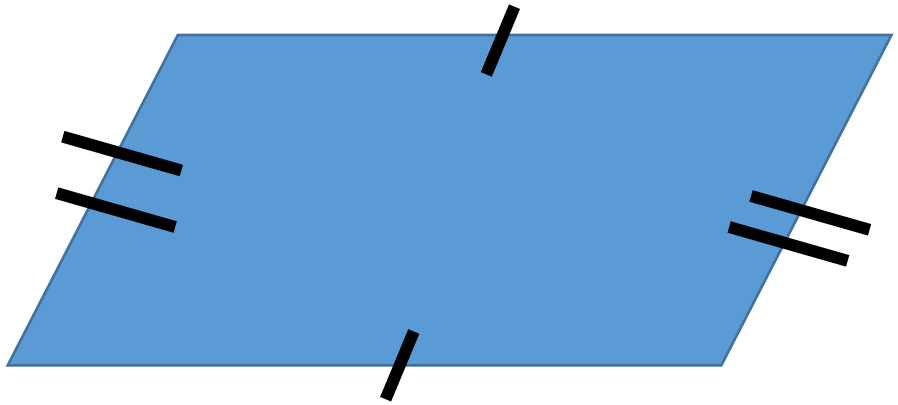
Now what do you know for sure?



Divergent math. It might be a parallelogram, but only if….

Geometric Figures

Are you any surer now about what you notice?



It must be a parallelogram because….

Visual Patterns

Here is what is great, but typical:

What might we ask?

Visual Patterns - alternatively

What do you notice? See if you and your partner can find 5 things.

If this is stage 2, draw and describe stage 1. How about stage 3?

Visual Patterns

Who has this? Who has something different?

What might we ask?

So we get to the same place, but with greater ownership and interest.

Adapted Freyer Model

Givens	**Picture or Model**
Numbers and/or Symbols	**Solution with Justification**

Strategies for supporting struggling students

In addition to gradual release and cumulative review and context:

- Multiple representations
- Alternative approaches

Key Mindset

In mathematics fundamentally:

- **A set of rules to be learned and memorized to find answers to exercises that have limited real world value**

OR

- **A set of competencies and understanding driven by sense-making and used to get solutions to problems that have real world value**

Universal Design for Learning Guidelines

I. Provide Multiple Means of Representation

1. Provide options for perception

- Options that customize the display of information
- Options that provide alternatives for auditory information
- Options that provide alternatives for visual information

2. Provide options for language and symbols

- Options that define vocabulary and symbols
- Options that clarify syntax and structure
- Options for decoding text or mathematical notation
- Options that promote cross-linguistic understanding
- Options that illustrate key concepts non-linguistically

3. Provide options for comprehension

- Options that provide or activate background knowledge
- Options that highlight critical features, big ideas, and relationships
- Options that guide information processing
- Options that support memory and transfer

II. Provide Multiple Means of Action and Expression

4. Provide options for physical action

- Options in the mode of physical response
- Options in the means of navigation
- Options for accessing tools and assistive technologies

5. Provide options for expressive skills and fluency

- Options in the media for communication
- Options in the tools for composition and problem solving
- Options in the scaffolds for practice and performance

6. Provide options for executive functions

- Options that guide effective goal-setting
- Options that support planning and strategy development
- Options that facilitate managing information and resources
- Options that enhance capacity for monitoring progress

III. Provide Multiple Means of Engagement

7. Provide options for recruiting interest

- Options that increase individual choice and autonomy
- Options that enhance relevance, value, and authenticity
- Options that reduce threats and distractions

8. Provide options for sustaining effort and persistence

- Options that heighten salience of goals and objectives
- Options that vary levels of challenge and support
- Options that foster collaboration and communication
- Options that increase mastery-oriented feedback

9. Provide options for self-regulation

- Options that guide personal goal-setting and expectations
- Options that scaffold coping skills and strategies
- Options that develop self-assessment and reflection

CAST

APA Citation: CAST (2008). *Universal design for learning guidelines version 1.0.* Wakefield, MA: Author.

So what is a more teacher-friendly way to say all of this?

Join me in Teachers' Chat Room

- **They forget**
- **They don't see it my way**
- **They approach it differently**
- **They don't follow directions**
- **They give ridiculous answers**
- **They don't remember the vocabulary**
- **They keep asking why are we learning this**

THEY THEY THEY BLAME BLAME BLAME

An achievement gap or an INSTRUCTION gap?

Well…..if…..

- **They forget – so we need to more deliberately review;**
- **They see it differently – so we need to accommodate multiple representations;**
- **They approach it differently – so we need to elicit, value and celebrate alternative approaches;**
- **They give ridiculous answers – so we need to focus on number sense and estimation;**
- **They don’t understand the vocabulary – so we need to build language rich classrooms;**
- **They ask why do we need to know this – so we need to embed the math in contexts.**

Adding and Subtracting Integers

Remember How

5 + (-9)

"To find the difference of two integers, subtract the absolute value of the two integers and then <u>assign</u> the sign of the integer with the greatest absolute value"

Understand Why

5 + (-9)

- **Have $5, lost $9**
- **Gained 5 yards, lost 9**
- **5 degrees above zero, gets 9 degrees colder**
- **Decompose 5 + (-5 + -4)**
- **Zero pairs: x x x x x O O O O O O O O O**
  - **On number line, start at 5 and move 9 to the left**

Solving Simple Linear Equations

$3x + 7 = 22$

How do we solve equations:

Subtract 7

$$\begin{array}{rcr} 3x + 7 & = & 22 \\ -7 & & -7 \\ \hline 3x & = & 15 \end{array}$$

Divide by 3

$$\frac{3x}{3} = \frac{15}{3}$$

Voila:

$$x = 5$$

3x + 7

1. **Tell me what you see: 3 x + 7**
2. **Suppose x = 0, 1, 2, 3…..**
3. **Let's record that:**

x	3x + 7
0	7
1	10
2	13

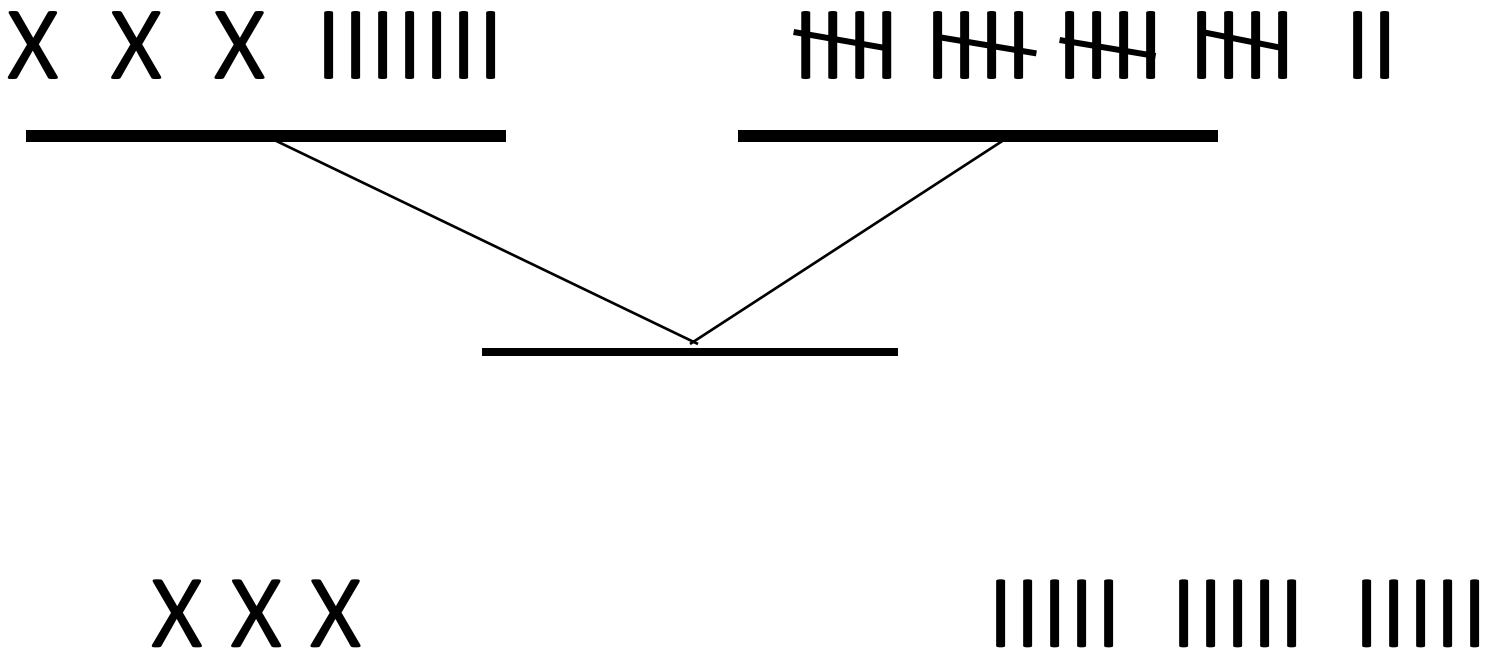
4. How do we get 22?

$3x + 7 = 22$

Where did we start? What did we do?

	x	**5**	
x 3	**3x**	**15**	**÷ 3**
+ 7	**3x + 7**	**22**	**- 7**

$$3x + 7 = 22$$



Let’s look at a silly problem

Sandra is interested in buying party favors for the friends she is inviting to her birthday party.

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Let’s look at a silly problem

Sandra is interested in buying party favors for the friends she is inviting to her birthday party. The price of the fancy straws she wants is 12 cents for 20 straws. The storekeeper is willing to split a bundle of straws for her. She wants 35 straws. How much will they cost?

So?

Your turn. How much?

How did you get your answer?

I solved it by making a table
here's an exsample

Table

Money	Straws
3 ¢	5
6 ¢	10
9 ¢	15
12 ¢	20
15 ¢	25
18 ¢	30
21 ¢	35

It will cost 21 ¢ for 35 straws

¢

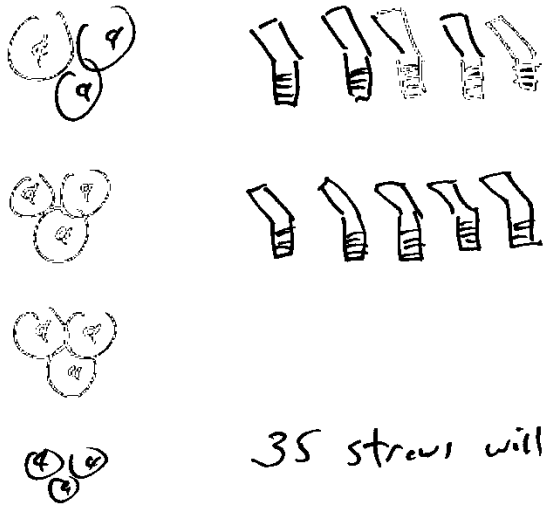
¢6 ||||||||||

¢6 ||||||||||

¢6 ||||||||||

¢3 ||||| ¢21

I solved it by drawing a picture



35 straws will cost 21 ¢

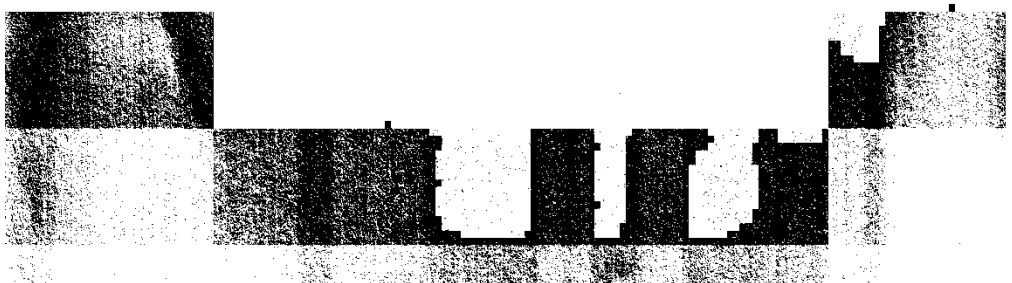
You want 10 straws you pay 6 cents
and for 3 ~~straws~~ cents you get 5 straws, so
there are 3 tens so you multiply 6×3
that's 18 & there's 5 left so I
added 3 cents & that makes 21 ¢

I divided 20 by 4. It equled 5
Then I did 12 ÷ 4. I equled 3

5 straws for 3¢. I carried
out the rat

5	10	15	20	25	30	35
3¢	6¢	9¢	12¢	15¢	18¢	21¢

The answer is 21 ¢



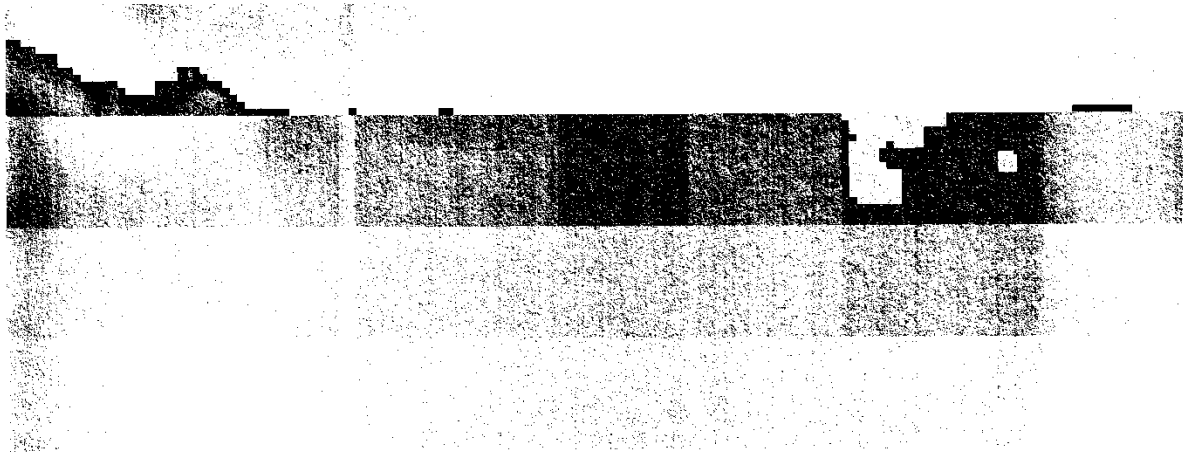
20 straws — 12 a

35 straws — x

$$\frac{x \cdot \cancel{20}}{\cancel{20}} = \frac{35 \cdot 12}{20}$$

$x = 21$ a

$$\begin{array}{r} 35 \\ 12 \\ \hline 70 \\ 35 \\ \hline 420 \end{array} \qquad \begin{array}{r} 21 \\ 20\overline{)420} \\ 40 \\ \hline 20 \\ 20 \\ \hline 00 \end{array}$$



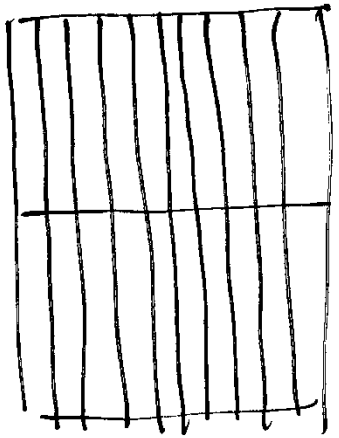
20 | 10 | 5

35

$$\begin{array}{r} 12 \\ 6 \\ +\ 3 \\ \hline 21\text{¢} \end{array}$$

First I got 20 for one bundle.
Then I halfed one bundle. Then
I put a bundle in for the 4
Then I got the prices for each
one whole, one half + one fourth
and I com up with 21 cents

I did this by finding out what each straw cost by dividing 20 by 12 on a calculator and ~~I~~ got 0.6 which is 6/10 then I multiplied 6/10 by 35 on a calculator and got 21 q



$\frac{6}{10}$ $\frac{6}{10} = \frac{12}{20}$

$= 20$

$\frac{12}{20} \begin{matrix} +6 \\ +10 \end{matrix} = \frac{18}{30} \begin{matrix} +3 \\ +5 \end{matrix} = \frac{21}{35}$

First you get 2 = 10's = 20

So that equals $= \frac{12}{20}$

then $\frac{+6}{+10} = \frac{18}{30}$ then

half of 10 equals 6 = 3 and

18 + 3 = 21

Strategy #4

Draw pictures/
Create mental images/
Foster visualization

The power of models and representations

Siti packs her clothes into a suitcase and it weighs 29 kg.

Rahim packs his clothes into an identical suitcase and it weighs 11 kg.

Siti's clothes are three times as heavy as Rahims.

What is the mass of Rahim's clothes?

What is the mass of the suitcase?

The old (only) way:

Let S = the weight of Siti's clothes

Let R = the weight of Rahim's clothes

Let X = the weight of the suitcase

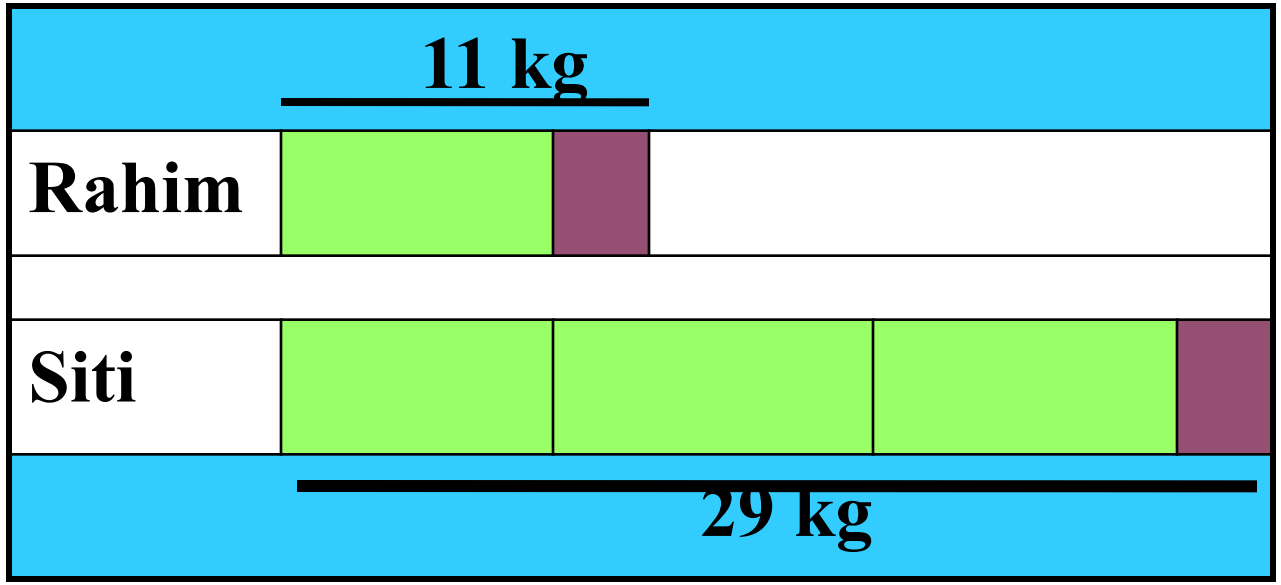
$$S = 3R \qquad S + X = 29 \qquad R + X = 11$$

so by substitution: $3R + X = 29$

and by subtraction: $2R = 18$

so $R = 9$ **and** $X = 2$

Or using a model:



Back where we started:

Josh and his family went to the carnival. It costs $5 to park the car plus $1.50 for each ride.

Josh's family spent a total of $38. How many rides did they go on?

- **Symbols**
- **Tables**
- **Bars**
- **Graphs**

Your turn

Select a concept or skill you will be teaching in the next four weeks.

Wrestle with alternative approaches and multiple representations that might support struggling students.

Strategies for supporting high flyers

- Enrichment (see released SAT problems, Math Counts problems, The Art of Problem Solving, Balanced Assessment, Mathematics Assessment Project) in the form of going deeper, multiple solution paths, clear explanations implemented via a 2 + 1 or 3 + 1 strategy
- Everything in math moves from A to B, use problems based on “inverse” that move from B to A

“If you’re so smart:

- **Challenge, not routine**
- **Multiple approaches, not one method or one solution path**
- **Multiple representations, not just symbols**
- **Justifications, not just answers**
- **Oral, not just written**

CCSSM Mathematical Practices (the first four)

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **<u>Construct viable arguments and critique the reasoning of others.</u>**
4. **Model with mathematics.**

Isn't this what our brightest students will need most in a world of google, symbol manipulators and powerful computers???

Ready!

At Central Middle School, there are 3 students in the Science Club for every 8 students in the Math Club.

If there are a total of 45 students in one or both of these clubs, how many scholars are in both clubs?

Be prepared to explain your reasoning.

And we're off….

- **Why is this a cool problem?**
- **Why is this a challenging problem?**
- **Why is this an appropriate problem for able students?**
- **Why would I accelerate when there is so much enriching that can be done?**
- **What is really needed in the world of work and citizenship? Skills a year earlier or the reasoning and problem solving capacities so often overlooked?**

Thinking vs. Math from the SAT

24. The number of boys attending Fairfield High School is twice the number of girls.

If 1/6 of the boys and 1/4 of the girls are in the school band, what fraction of the student are in the school band?

5/36 7/36 2/9 7/24 5/12

The “right” way:

Let B = the number of boys

Let G = the number of girls

$$2G = B$$

Find: $$\frac{1/6\ B + 1/4\ G}{B + G} = \frac{1/6\ (2G) + 1/4\ G)}{2G + G}$$

$$= (2/6\ G + 1/4\ G)/3G$$

$$= 7/12\ G\ /\ 3G = 7/36$$

The Stanley Kaplan Approach

Boys		Girls
2	:	1
1/6		1/4

Try: 100 students – 2 to 1 – no

Try: 90 students – 60 and 30, 6 and oops

Try: 6ths & 4ths – 24 and 48 – 8 + 6 out of 72

14/72 = 7/36

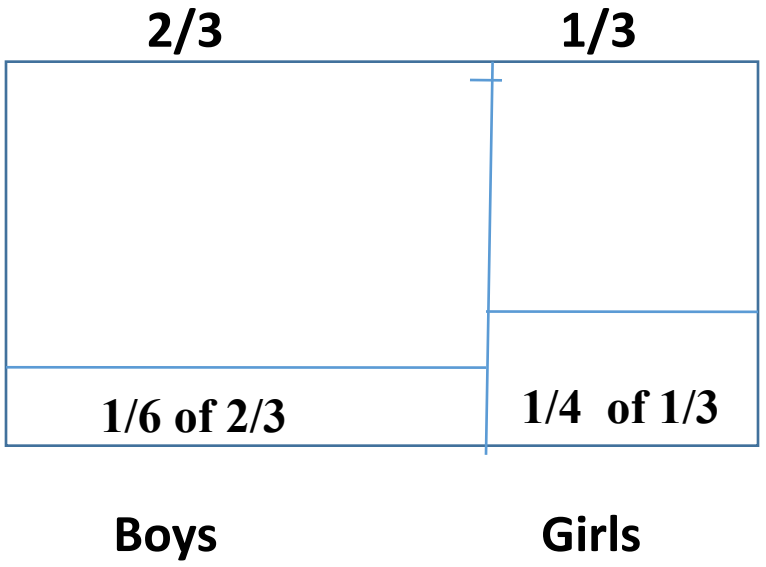
Or play, beauty, joy

K kids, ergo boys = 2/3K and girls = 1/3K

Band = 1/6 (2/3K) + 1/4 (1/3K)

= (1/9 + 1/12) K or 7/36 K

OR



Thoughts on the high flyers

- **Deeper and enriched is far more appropriate than faster and accelerated, despite parents convictions about the giftedness of their children and the fact that accelerating is so much easier to do. (Severely gifted and in need of acceleration about 1 in 300)**
- **Going deeper and enriching are must harder to do than merely accelerating.**
- **Everyone benefits when the high flyers remain heterogeneously grouped with less able students.**

Mathematics??? Deeper Learning??? In 2019???

$$\frac{45}{\sqrt{2} + \sqrt{7}}$$

Convince us that this still has a role in the high school curriculum.

Mathematics??? Deeper Learning??? In 2019??

Kai claimed that there are only three regular polygons that tessellate the plane.

His sister asked: Which three?

His mother asked: Why only those three?

Provide answers that Kai can give his sister and this mother. Explain why a regular octagon does not tessellate the plane.

Forward and BACKWARDS

A function and its inverse.

Here is independent variable, find dependent. Easy. Now here is dependent variable, find independent. (Evaluate vs. Solve)

Here is situation and data, find answer. Here is answer, find conditions.

Your turn

Pencils 7¢

Pens 8 ¢

Erasers 9 ¢

Limit of 10 of each.

SO?

Change from $1.00 if you buy…..

Vs.

I just spent 83¢ (no tax) in this store.

What did I purchase?

“Inverse” problems

- **Create a set of 7 two-digit numbers whose median and mean are the same.**
- **Create a two-step equation whose solution is x = 9.**
- **Create a system of linear equations whose solution is (-4, 3).**
- **MORE**

Your turn

With a partner, create a turnaround, that is, from routine to non-routine so as to enrich the lesson.

How much is: $8/5 \div 0.7$

- **More than 2 or Less than 2? How do you know?**
- **Exactly how much?**
- **Turn and tell your partner what you think the answer is?**
- **(Collect answers)**
- **Can they all be correct?**
- **Turn and convince your partner which answer is correct?**
- **Who can explain?**
- **Who did it differently?**
- **What did you just learn?**

Task Tweaks: What is 18 ÷ 3???

Awesome:

- **Write that as multiplication.**
- **Show your answer as equal groups.**
- **Show your answer as an array or area model.**
- **Show where your answer fits into a multiplication table.**
- **Write a word problem that requires you to divide 18 by 3.**

Awesome: What about 19 ÷ 3?

Tweaking their brains:

1. Write a story problem that requires a normal human being to find 8/5 divided by 0.7.
2. Create a model – convince someone that your model works
3. If you did this with fractions, do this with decimals, and vice versa
4. Explain orally and in writing
5. 7/6 divided by 0.15
6. Our original problem had an answer a little more than 2.

   change the 8/5 so that the answer is less than 1

   change the 0.7 so that the answer is less than 1

Reading/Discussion Break

Why SMP #3 might just be the 9 most important words in the Common Core.

Three minutes of individual reflection. Then sharing with a partner.

- **What resonates with me the most from these two pages is……**
- **What I'm not sure about is……**
- **How much CVA&CRO would you find in my classes? Why?**

Enrichment Ideas

In other words:

Given a set of data:

- **Organize the data, identify trends, ask questions, develop and justify hypotheses, summarize the data**

Given a situation:

- **Represent it, describe it, interpret it, make predictions, plan a course of action, decide what's fair, identify contradictions or rip-offs**

In other words:

Given a claim:

- **Evaluate it**

Given a set of constraints:

- **Satisfy them, find an optimum solution**

Given a model:

- **Explain it, debug it, refine it, generalize it, evaluate it, apply it**

****Notice how this aligns with the CCSSM Standards for Mathematical Practice****

But the best reasons I have seen for why our highly able students need more than just acceleration……. or teaching the whole student:

Mathematics for Human Flourishing (adapted from Francis Su, 1/6/17 Joint Math Meetings Presidential Address)					
Desires	**Play**	**Beauty**	**Truth**	**Justice**	**Love**
Virtues	Community Hopefulness Perseverance	Joy Wonder	Rigorous thinking Humility Circumspection	Dignity Equity Fairness	Compassion Empathy
Flourishing	You cannot flourish with play	You cannot flourish without the joy and wonder of beauty	You cannot flourish with falsehoods	You cannot flourish with injustice	You cannot flourish without loving and being loved
Mathematics	Math is a playground of the mind!	Math is beautiful!	Math is the essence of truth!	Math can and must create social justice!	For all these reasons, math should be loved!
Conclusion	Yes, we teach our students the skills, concepts and applications of mathematics, but we also teach human beings who possess the human desires to play, and for beauty, truth, justice and love in their lives. What a joy that mathematics, well taught and with this shift in mindset, provides such rich opportunities to meet these human desires, attend to the virtues of a live well-lived, and foster intellectual, social and emotional flourishing in and outside of our classroom.				

Finding great short tasks:

Released PISA items -
http://www.oecd.org/pisa/38709418.pdf
Grades 8 and 10 MAA Mathematics Content
http://www.maa.org/math-competitions/amc-8
Math Counts released items
SAT Practice Tests –
https://collegereadiness.collegeboard.org/sat/practice/full-length-practice-tests
What else do you use?

Tasks that Support the Needs of Highly Able Students

(Let’s play!)

Moving people to equate density

South Dakota and North Dakota rank 45 and 47 in population of all the states. South Dakota has 721,000 people in 75,896 sq. miles. North Dakota has 638,000 people in 68,994 sq. miles.

Which state has the greater pop. Density?

How many people in one state would have to move to the other to make the pop density of the two states equal?

Pre-Calculus Polynomial Review Lesson

Which leads to a much more varied review

Mathematics: Answers vs. Answers AND Understanding

Pre-Calculus Polynomial Review Lesson

Context: The night before 1 to 16 review problems like:

5. Find all the zeros of $P(x) = x^3 - 5x^2 - 7x + 51$ if $(4 - i)$ is one of the possible zeros.

6. List all the possible rational roots of $P(x) = 4x^4 - 13x^3 - 13x^2 + 28x - 6$ and find them.

7. Show that 3/2 is a double root of $P(x) = 4x^4 - 12x^3 + 13x^2 - 12x + 9$.

8. Find the number of times that – 1 is a root of $P(x) = x^5 + 3x^4 + 2x^3 - 2x^2 - 3x - 1$

9. Find a polynomial with integral coefficients that has roots 3, $1 + 2i$.

10. Solve $2x^4 - 3x^3 - 3x - 2 = 0$; given that 2 and -1/2 are two of the roots.

6.) [CT?]Determine if each of the following scenarios is possible or not. **Then <u>explain</u> whey each is or is not possible.**

a) An even polynomial has no *y-intercept.*

___Possible

___Not Possible

b) An odd polynomial has all imaginary roots.

___Possible

___Not Possible

c) Three of the four roots of a degree four polynomial are -2, $3i$, and $1-4i$.

___Possible

___Not Possible

d) A rational function has no vertical asymptotes.

___Possible

___Not Possible

e) An odd polynomial has no *x-intercept.*

___Possible

___Not Possible

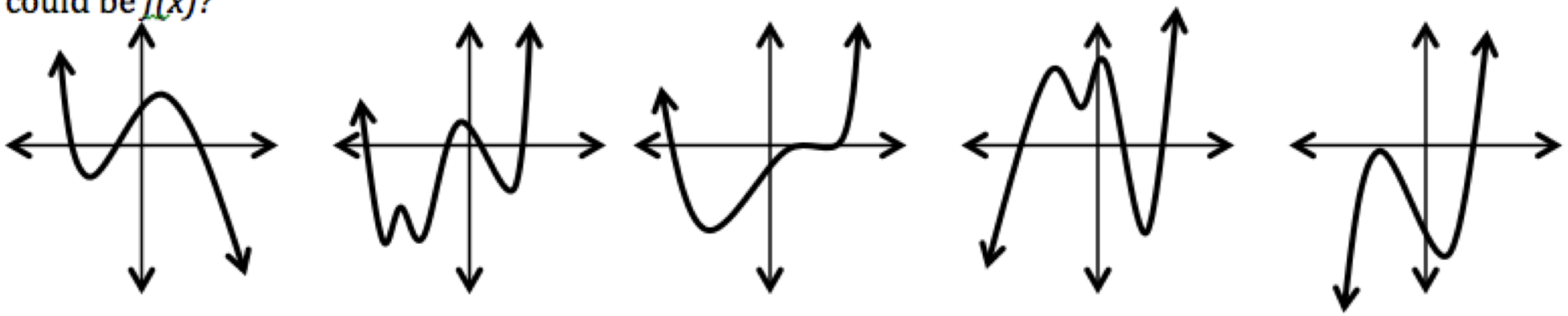
f) If $3-i$ is a root of $j(x)=ax^4+bx^2-cx+2i$ where $a,b,c\in\mathbb{R}$, then $3+i$ is also a root.

___Sometimes

___Always

___Never

7.) [K/U] Polynomial *f(x)* is of degree 3 and has a leading coefficient of 4. Which of the following graphs could be *f(x)?*



a) b) c) d) e)

1.) [AP] The polynomial $g(x)$ is of degree $n \geq 1$. If the polynomial $g(x)$ has only one real root with a multiplicity of one, what **must** be true about $g(x)$? Explain your answer to receive credit.

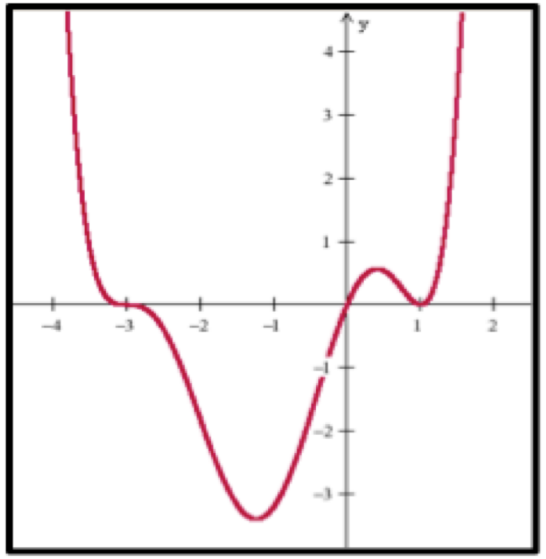
4.) [AP] On which of the following intervals does $g(x) = x^4 - 17x^2 + 30$ have a zero? Justify or explain all answers to receive credit.

a) $-1 < x < 1$ | b) $1 < x < 2$ c) $2 < x < 3$

8.) [AP] The graph of *f(x)* is show below.

a) If *f(x)* is of degree 6, find all zeros and determine their multiplicity.

b) If *f(x)* passes through the point (-1, -3), find the exact equation of *f(x)*.



What’s the story ?

JOANNE HOYOUNG LEE / THE HARTFORD COURANT

BEFORE THE PARADE PASSES BY, Bob Becker, owner of Gotogo in Killingworth, sets up portable toilets next to Soldiers and Sailors Memorial Arch in Bushnell Park on Friday afternoon. Becker said that more than 100 portable toilets would be set up for the 200,000 people expected to watch the parade in Hartford today celebrating the University of Connecticut men’s national basketball championship. The victory parade starts at 11 a.m. on Trinity Street.

The caption:

Before the parade passes by. Bob Becker of Gotogo in Killingworth, sets up portable toilets next to Soldiers and Sailors Memorial Arch in Bushnell Park in Hartford on Friday. Becker said that more than 100 portable toilets would be set up for 200,000 people expected.

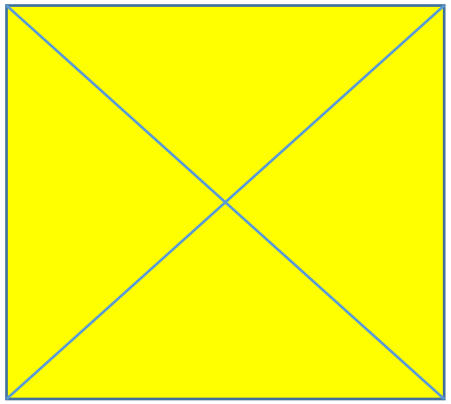
What’s the question?

- **How many people?**
- **How long a parade?**
- **Beer?**
- **Male/female balance?**
- **How many toilets are needed?**
- **Why should we care?**
- **Other?**

Fumi’s Squares

- Is this a square?
- How do you know?
- What will it take?
- What about. What about.

- Now:



Fumi’s Squares

- Is this a square?
- How do you know?
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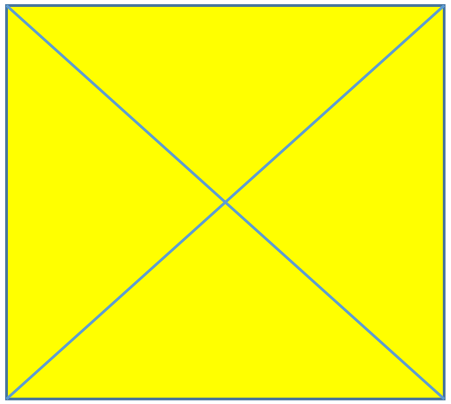
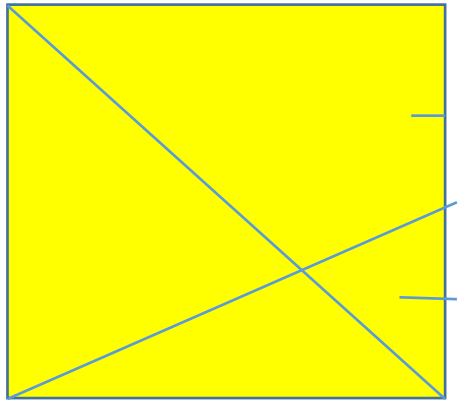
- What about. What about.

- Now:





Your turn – again with a same grade or course partner

- **What are you teaching this week or next?**
- **What are the big ideas that transcend the skills?**
- **What are one or two rich tasks that support the learning of this topic?**
- **How can you create a lesson (goals, tasks, questions) that provides enrichment and deeper learning opportunities for your more able students?**

- **This was fun because…..**
- **This was frustrating because…..**
- **This was engaging because…..**
- **This was worthwhile because….**

Resources?
Where can I find great stuff?
Let’s look together

www.steveleinwand.com/publications

Find a same grade colleague

- **Explore the resources looking for great or potentially great enriching tasks and activities.**
- **Select one or two.**
- **Convince yourself why you think it will help you better serve highly able students.**
- **Be ready to share your finding and argument.**

- **OKAY – We will return to these tasks/activities/problems at 2 p.m.**

Putting it all together

Two sample lessons – one middle school and one high school

Grade 6

Fractions Unit Review

Lesson Goal: To be sure we are comfortable multiplying and dividing with fractions

February 7, 2016

1. What does it mean to multiply fractions?

True or False:

$$\frac{3}{8} \text{ X } \frac{2}{3} = \frac{1}{4}$$

The rule tells us that to multiply two fractions……

What does it mean that the product of 3/8 and 2/3 is 1/4?

What if you forgot the rule, how do we find $\frac{3}{8}$ X $\frac{2}{3}$ **?**

2. What does it mean to divide fractions?

True or False:

$$1\frac{3}{4} \div \frac{2}{3} = 3\frac{1}{2}$$

The rule tells us that to divide two fractions……

What does it mean that the quotient of 1 3/4 and 2/3 is 3 1/2?

What if you forgot the rule, how do we find $1\frac{3}{4} \div \frac{2}{3}$ **?**

3. Do we multiply or divide?

Sara's cookie recipe requires 3/8 of a cup of sugar for each batch.

Sara has 10 cups of sugar in a jar.

What's the question?

What's a reasonable estimate? How did you make your estimate?

What's the operation you can use? Why?

Exactly how many batches can Sara make?

Can you draw a picture or diagram to convince yourself that your answer is correct?

4. Your turn

Make up a realistic problem that a normal human would solve by calculating

$$3\frac{1}{4} \text{ x } 2\frac{2}{3} =$$

Estimate and justify

Calculate

Convince

5. Your turn once again

Make up a realistic problem that a normal human would solve by calculating

$$9\frac{1}{2} \div \frac{3}{4} =$$

Estimate and justify

Calculate

Convince

6. Why does the rule work?

Turn and tell your partner WHY “invert and multiply” works to divide fractions.

7. Sentence completion

The biggest or most troublesome question I have about multiplying and dividing fractions is…….

Turn and tell your partner what you learned in the math lesson today.

Exit Ticket

What I need to do to be sure I do well on this unit test is…..

Good Morning Geometry!

Dilation: A Special Case of Similarity

Monday, October 17, 2016

Dilation, to Dilate something

- **What does it mean to dilate?**
- **What does NOT change when something is dilated?**
- **What DOES change when something is dilated?**
- **How is dilation related to similarity?**

Scale Drawings

What are scale drawings?

Are scale drawings dilations? Why? Why not?

What is the prominent feature of a scale drawing?

Yes, so what is the “scale” in a scale drawing?

So what do you see?

What’s changed?

How has Teddy grown?
Can we find the scale factor?

With a partner:

Draw a square. Label it Figure 1.

Dilate your square by a scale factor of 0.5 and label it Figure 2.

Dilate your square by a scale factor of 2 and label it Figure 3.

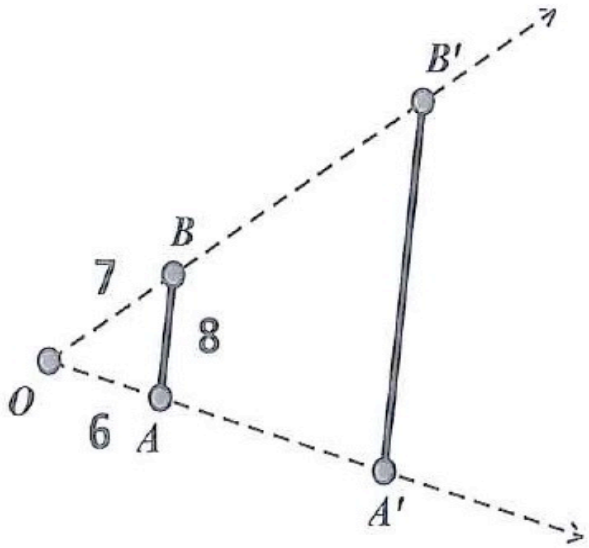
Compare the changes in side lengths and areas of the three squares.

The amazing thing about dilations:

Let’s line up our Teddy bears again.

Let’s line up our squares again.

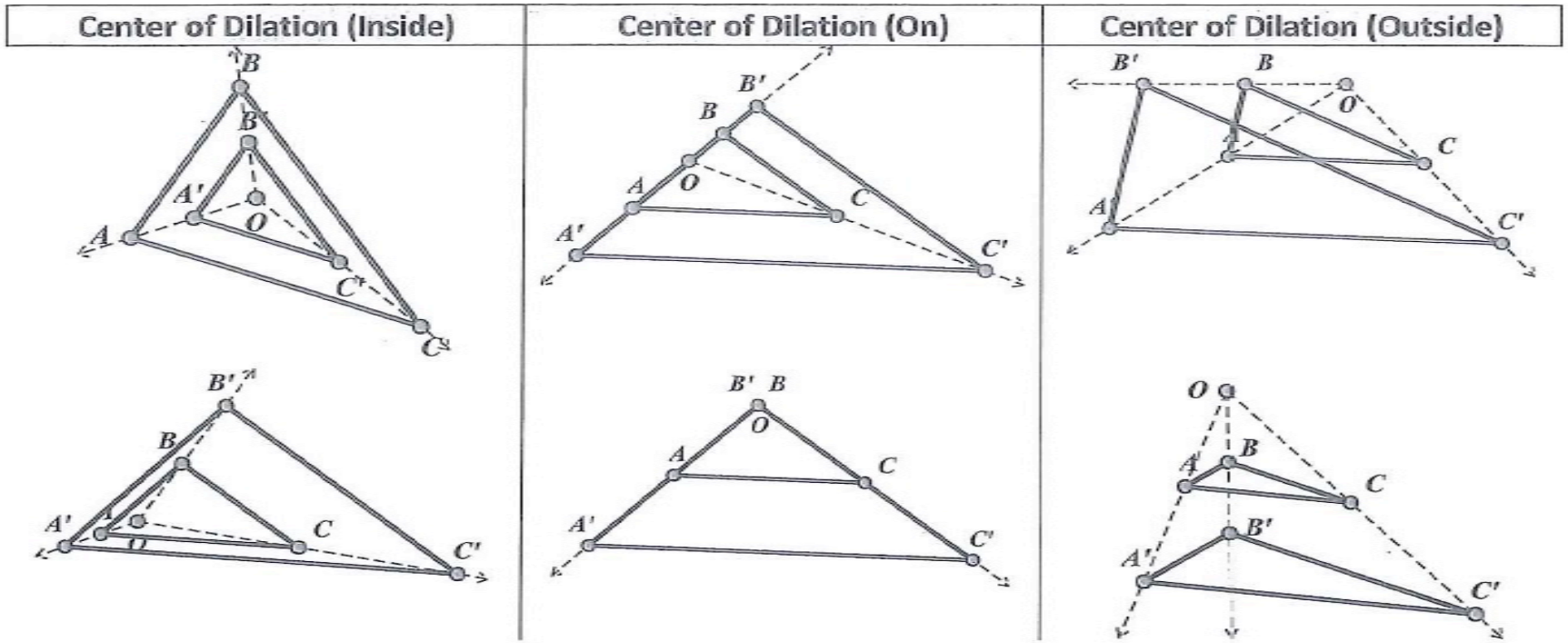
Using rays to represent and create dilations:



What do you notice?
What do you wonder?

Suppose A’B’ = 24.

But here's the fun part.
Dilations have centers. We dilate about a point.
Discuss with a partner what you see here.



Are you ready to do some dilations of triangles on graph paper??

Given triangle ABC, with A (0,0), B (3,1) and C (2,3), dilated with a scale factor of 3 about point O (0,3), find the coordinates of the resulting triangle A'B'C'.

A small change:

Given triangle ABC, with A (0,0), B (3,1) and C (2,3), dilated with a scale factor of 0.5 about point O (2,2)find the coordinates of the resulting triangle A’B’C’.

What did you learn today?

Exit slip

Draw rectangle ABCD. Now draw rectangle A’B’C’D’ that results from a dilation of ABCD with a scale factor of 1/3 about vertex B.

Summary and Personal To-do Lists to Follow-up the Workshop

Processing Questions

- **What are the two most significant things you've heard in this presentation?**
- **What is the one most troubling or confusing thing you've heard in this presentation?**
- **What are the two next steps you would support and work on to make necessary changes?**

Next Steps

People won't do what they can't envision,

People can't do what they don't understand,

People can't do well what isn't practiced,

But practice without feedback results in little change, and

Work without collaboration is not sustaining.

Ergo: Our job, as professionals, at its core, is to help people envision, understand, practice, receive feedback and collaborate.

To collaborate, we need time and structures

- **Structured and focused department meetings**
- **Before school breakfast sessions**
- **Common planning time – by grade and by department**
- **Pizza and beer/wine after school sessions**
- **Released time 1 p.m. to 4 p.m. sessions**
- **Hiring substitutes to release teachers for classroom visits**
- **Coach or principal teaching one or more classes to free up teacher to visit colleagues**
- **After school sessions with teacher who visited, teacher who was visited and the principal and/or coach to debrief**
- **Summer workshops**
- **Department seminars**

To collaborate, we need strategies 1

Potential Strategies for developing professional learning communities:

- **Classroom visits – one teacher visits a colleague and the they debrief**
- **Demonstration classes by teachers or coaches with follow-up debriefing**
- **Co-teaching opportunities with one class or by joining two classes for a period**
- **Common readings assigned, with a discussion focus on:**
  - **To what degree are we already addressing the issue or issues raised in this article?**
  - **In what ways are we not addressing all or part of this issue?**
  - **What are the reasons that we are not addressing this issue?**
  - **What steps can we take to make improvements and narrow the gap between what we are currently doing and what we should be doing?**
- **Technology demonstrations (graphing calculators, SMART boards, document readers, etc.)**
- **Collaborative lesson development**

To collaborate, we need strategies 2

Potential Strategies for developing professional learning communities:

- **Video analysis of lessons**
- **Analysis of student work**
- **Development and review of common finals and unit assessments**
- **What's the data tell us sessions based on state and local assessments**
- **"What's not working" sessions**
- **Principal expectations for collaboration are clear and tangibly supported**
- **Policy analysis discussions, e.g. grading, placement, requirements, promotion, grouping practices, course options, etc.**

The obstacles to change

- **Fear of change**
- **Unwillingness to change**
- **Fear of failure**
- **Lack of confidence**
- **Insufficient time**
- **Lack of leadership**
- **Lack of support**
- **Yeah, but…. (no money, too hard, won't work, already tried it, kids don't care, they won't let us)**

Finally – let’s be honest:

Sadly, there is no evidence that a session like today makes one iota of difference.

You came, you sat, you were “taught”.

I entertained, I informed, I stimulated.

But: It is most likely that your knowledge base has not grown, you won’t change practice in any tangible way, and your students won’t learn any more math.

And this is what we call PD.

Prove me wrong

by

Sharing

Supporting

Taking Risks

Next steps: Taking Risks

It all comes down to taking risks

While “nothing ventured, nothing gained” is an apt aphorism for so much of life, “nothing risked, nothing failed” is a much more apt descriptor of what we do in school.

Follow in the footsteps of the heroes about whom we so proudly teach, and TAKE SOME RISKS

Thank you!